EXACT SCIENCES FROM JAINA SOURCES

VOL.-1

BASIC MATHEMATICS

RAJASTHAN
PRAKRIT BHARATI SANSTHAN
JAIPUR

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EXACT SCIENCES FROM JAINA SOURCES

Vol.-1

BASIC MATHEMATICS

by

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Forward

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Rajasthan Prakrit Bharti Sansthan, Jaipur, has many streams of publications; it brings out books on philosophies, religions their comparative studies, history, art & culture, ancient literary works, Prakrit language and secular scientific subjects etc. On the last one presently we propose to bring out a series of ten books in collaboration with renowned Sitaram Bhartia Institute of Scientific Rerearch, New Delhi, out of which this one is the first.

We are indeed grateful to Professor L. C. Jain, an eminent authority on history of mathematics, to have written this book which might be useful in appreciating the contribution of ancient scholars to the development of mathematics.

We are also indebted to Dr. G. C. Patni formerly Professor and Head of Department of Mathematics and Dean, Faculty of Science, University of Rajasthan, Jaipur for contributing the forward of the book.

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FOREWORD

Ancient India has contributed a lot to the development of mathematics and the part played by the Jaina scholars in this field is very significant. These contributions are far superior to those made by other nations and countries upto the 12th century A. D. or so. In ancient India, Mathematics and Astronomy were given a very high place in the field of knowledge as is evident from the following verse of Vedāṅga :

यथा शिखा मयूराणां, नागानां मणयो यथा ।
तद्वद् वेदाङ्ग शास्त्राणां, गणितं मूर्धनि स्थितम् ।।

This when translated into English would mean : "As are crests on the heads of peacocks, as are gems on the heads of snakes, so is Mathematics at the top of all sciences—branches of knowledge."

Amongst the six fundamental branches of knowledge, viz.

(i) शिक्षा —The Science of proper articulation and pronunciation–phonetics,

(ii) कल्प —Rules for rituals and ceremonials (śulva-Sūtra supplement to Kalpa Sūtra),

(iii) व्याकरण —The Science of right use of language,

(iv) निरुक्त —Etymology (The science of origin of words),

(v) छंद —The Science of prosody (Metronomy),

(vi) ज्योतिष —Astronomy—the Science of stars and other heavenly bodies,

Astronomy occupies a prominent place as mentioned in another verse :

शिक्षा कल्पो व्याकरणं, निरुक्तं छन्दसा चैव ।
ज्योतिषानयनं चैव, वेदांगानि षडैव तु ।।

In fact all knowledge can be divided into two main streams (i) the science of letters (अक्षर विद्या) (ii) the science of numbers (अंक विद्या). In the former are included subjects like Grammar, Literature, Logic etc. while in the latter are included Gaṇita, Astronomy, Science, Economics, Commerce etc.

But as we all know now and as the great Jaina mathematics scholar Mahāvīrācārya (850 A. D.) had said even more than eleven hundred years ago, gaṇita is used in almost all branches—subjects of knowledge for their perfect and mature study. Looking to the importance of mathematics in ancient India, even the Jain literature was divided into four main classes : (i) Prathamānuyoga (प्रथमानुयोग) which includes purāṇa, stories, descriptive books, biographies etc., (ii) Karaṇānuyoga (करणानुयोग) which includes literature on astronomy, mathematics (the science of measurement and calculation), etc. (iii) Caraṇānuyoga (चरणानुयोग) which includes the rules, sub-rules to be followed by saints, sages, śrāvakas etc. and (iv) Dravyānuyoga (द्रव्यानुयोग) which includes the description of fluents-elements like jīva (जीव–living), ajīva (अजीव–non-living) etc. Our knowledge of the history of Indian Mathematics prior to Āryābhaṭa (498 A. D.) is very imperfect; whatever is available is through Jaina mathematics contained in their religious texts. But our knowledge of even these is at present very limited and it is felt that as more and more of Jaina works and other ancient Hindu and Budhist writings are discovered, more and more of fresh light on the contributions of ancient India to mathematics, as in other fields of knowledge, would be thrown. Even the Bhakhṣāli manuscript found in Bakhṣālī village near Peshāwar in 1881 and containing the description of a number of arithmetical and algebraic operations is said to belong to 12th century A. D., though according to some scholars, it may belong to 3rd or 4th century A. D. Further the work 'Āryāṣṭaśta' (Ārya-Bhaṭīya) contains only one independent chapter consisting of 33 verses on Mathematics. Similarly the other great mathematician Brahmagupta (c. 628 A.D.) has added only one chapter on mathematics in his book 'Brāhmasphuṭa Siddhānta.' It appears that the text, 'Gaṇitasāra Saṁgraha' by the Jaina scholar Mahāvīrācārya (c.850 A. D.) is perhaps the first book wholly dealing with mathematics. However long before Mahāvīrācārya, the Indian mathematicians including the Jainas had developed mathematical sciences which in turn helped in the development of many other subjects of science, arts, humanities and social sciences—a fact which Mahāvīrācārya has himself acknowledged in the following words,* "With the help of the holy accomplished sages who are worthy to be worshipped by the lords of the world and of their disciples and disciples' disciples who constitute the well known jointed series of preceptors, I glean from the great ocean of the knowledge of numbers, a little of its essence, in the manner in which gems are picked up from the sea, gold from the strong rock and the pearl from the oyster shell and give out according to the power of intelligence, the Sāra-saṁgraha, a small work on arithmetic which is, however, not small

*See also the Text, Chapter 1.

in value." He further remarks, "In all transactions which relate to worldly, vedic or other similar religious affairs or in the science of love, in the science of economics, in music or in drama, in the art of cooking, in medicine, in architecture, in prosody, in poetry, in logic and grammar and such other things and in relation to all that constitutes the peculiar value of the arts, the science of calculation (in modern technology mathematics) is held in high esteem. Not only this, but in relation to the movement of the sun, the moon and other heavenly bodies, in all astronomical phenomena whether they be related to eclipses, conjunction of planets, tripraśna (direction, position and time), the measurements of small or extensive dimensions of lengths and heights of small or big bodies like atoms, islands, ocean and mountains, interspace distances between the different worlds, the world of gods (heavens), the world of hill, that is, in all spheres of life which the man can think of, gaṇita is used. 'The configuration of living beings therein, the lengths of their lives, their progress, their staying together, that is, in other words, whatever there is in all the three worlds (tri loka) which consist of moving and non-moving beings, cannot exist apart from gaṇita." In fact Mahāvīrācārya has said :

छंदो ऽलंकार काव्येषु, तर्कं व्याकरणादिषु ।
कला गुणेषु सर्वेषु, प्रस्तुतं गणितं परम् ॥

meaning thereby that gaṇita is at the top of all branches of knowledge. Such was the importance of gaṇita (the science of measurement and calculation) recognised by Mahāvīrācārya even during those old days. During the modern times also, looking to the importance of mathematics and its applications to all other abstract sciences, the great social philosopher Comte has placed mathematics as first in the heirarchy when considering certain subjects like mathematics, physics, chemistry, biology, sociology, etc. Taking into consideration the recent researches in the various fields of knowledge,their heirarchy in the modified form can be arranged roughly as : Mathematics-Statistics-Physics-Engineering and Technology-Chemistry-Biology including medical Sciences - Agriculture - Economics - Psychology - Sociology- Political Science-Earth Sciences-Literature-History. In this heirarchy each science is historical, 'older, logically simpler, more widely applicable, more capable of reaching the positive developed state earlier and independent of all those that follow it. Thus in order also, mathematics occupies the first place.

As in someother parts of the world, in India also the stream of spiritual thinking (आध्यात्मिक विचारधारा) has been going on for the last thousands of years. Through this spiritual thinking only, the man has achieved his spiritual progress. Many preceptors have worked in this field

and guided the man, e. g., Lord Ṛṣabhadeo, Lord Rāma, Lord Kṛṣna, Lord Pārśvanātha, Lord Mahāvīra, Lord Buddha etc. There have been a lot of similarities as well as dissimilarities in the teachings of these spiritual leaders. But all have worked in their own manner for the welfare of the human kind. The efforts made by Lord Mahāvīra and his followers for achieving love and affection towards all, unity in diversity through the great principles of Ahiṃsā (non-violence) and Syādvāda and self-attainment are worth mentio, ning. As a matter of fact, in India the aims of most of the philosophies (darśanas), specially of the Jaina Philosophy, have been mainly :

(i) Ātma tṛpti (आत्म तृप्ति), (ii) Ananta tṛpti (अनंत तृप्ति)
(iii) Swarūpa prāpti, (स्वरूप प्राप्ति)

and for all these it is necessary that one should be able to concentrate on his own mind and ātmā (self-soul). The aims of mathematical teaching also are to address the mind and to check the outer.

Thus the possibility of having a close relationship between a sound philosophy and mathematics is natural and self-evident. Now for the study of cosmos and self, two worlds, one the macroworld and the other microworld, are to be manifested to a human intellect and with this object, in view, the Jaina scholars in ancient India have contributed a lot to Astronomy, Mathematics as well as to Jaina Karma Theory with mathematical approach. Some of the important works on basic mathematics now known include the 'Gaṇita Sāra-Saṃgraha of Mahāvīrācārya (c. 850 A. D.) While some other treatises relating to astronomical sciences are the Sūrya Prajñapti and the Candra-prajñapti of about 300 B. C.

The Jain Āgamas discuss, amongst other things, the Karma Theory in particular. In these Āgamas lie the deep secrets of periodic events of nature, involving ten types of infinities. It is unfortunate that many of the Āgamas in which mathematics has been developed either as post-universal (alaukika) study of measures and counting or applied in form of results through analysis and comparibility, e.g., the works of Bhadrabāhu who died in 278 B. C., either have been lost or are not traceable at present. Some of the important available Āgamas are the Kasāya-pāhuḍa, the Ṣaṭkhaṇḍāgama, the Tiloypaṇṇatti, their extracts like the Kṣapaṇāsāra (including Labhdisāra), the Gommaṭasāra and the Trilokasāra and their valuable commentaries : Dhavalā, Jayadhavalā, Mahādhavalā, Jīvatattva-pradīpikā and Saṃyakjñāna candrikā. Some of these original works are more than 2000 years old, but many of these lay hidden and could be brought to light only recently. Raṅgācāryā in 1912 found a copy of the Gaṇitasāra-Saṃgraha of Mahāvīrācārya and produced an English translation of it. It is only after 1912 that

active research work on Jain School of Mathematics started; credit for this goes to authors like Jain (H. L.), Upādhyāya, Smith, Kāpadiyā, Singh, Datta, Saraswati, Jaina (L. C.), Guptā, Lishk and Sharmā, Volodarski etc. Even within this short span of time, there have been discovered certain mathematical results and principles which are not only highly interesting but are also considered very important and useful for the study and exposition of ancient Indian Mathematics in general and the Jain mathematics in particular. Dr. A. N. Singh has also remarked that a study of the Jaina canonical works reveals that mathematics was held in high esteem by the Jainas and the knowledge of mathematics and astronomy was considered to be one of the principal accomplishments of the Jaina ascetics and scholars who have written apart from those listed above, works like Sthānāṅga-sūtra, Tattavārthādhigamasūtra, Sūrya-prajñapti, (Gaṇitānuyoga), Anuyogadvāra-sūtra, Jambūdvīpa-Prajñapti, Lokavibhāga, Bhagavatī-sūtra, Kalpasūtra, Candraprajñapti, Jyotiṣa-Karaṇḍaka etc. Though many of these works are non-mathematical in character but as recognised by Singh also, the knowledge of Jaina mathematics is gleaned very much from them. Most of these works belong to the period much before the 5th century A. D. Āryabhaṭa First (c. 498 A. D.), Brahmgupta (c. 628 A. D.), Mahāvīrācārya (c. 850 A. D.) and Bhāskarācārya (c. 1150 A. D.) are the most eminent known mathematicians of ancient India, though a number of other persons also have worked on mathematics. One of the most important fundamental contributions of ancient Indian mathematicians to the progress of civilisation is the invention of what is called the Decimal system of Numeration including the invention of the number "Zero" and a symbol for it, and the principle of local value which had revolutionised the system of counting and measurement and these were given thousand of years back by Indian mathematicians, their names still being unknown. The importance of the decimal system of numeration can best be appreciated in the words of foreign mathematicians. Laplace (1742-1827), one of the greatest mathematicians of all times says, "The idea of expressing all quantities by nine digits and a symbol for zero, whereby imparting to them both an absolute value and a positional (local) value, is so simple that this very simplicity is the reason for our not being sufficiently aware how much admiration it deserves." Prof. G. B. Halstead also remarks, "The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local inhabitation and a name, a picture, a symbol but also a helpful power is the characteristic of the Hindu race whence it sprang up. No single mathematical invention has been more potent for the general on-go of intelligence and power." What a greater tribute can be paid to the genius of the Indian mathematicians ? Prof. Halstead has also proved that the 'zero', existed in India at least at the time

of Pingala's work Chanda Sūtra—a work on prosody before 200 B. C.

It is said that the concept of infinity in mathematics was conceived for the first time by the Indian mathematician Brahmagupta (c. 628 A. D.) while in the western countries, this credit goes to Bernhard Bolzano of the 19th century A. D. A study of infinity in mathematics as a mature concept was however taken up by Bhāskarācārya (c. 1150 A. D. He appears to be the first mathematician to have deduced the value of the quotient $\frac{a}{0}$ mathematically where a is a finite quantity and termed it as ananta. But the description of infinity as endless or countless can be traced in Ṛgveda and many other ancient works including those by Jaina and Buddhist scholars. An elaborate classification and philosophical explanation of infinity (ananta) is however found in the Jaina canonical texts as old as 300–400 B. C. where infinity of even ten types has been mentioned, e. g. in the Sthānāṅga-Sūtra (325 B. C.), Uttarādhyana-Sūtra (300 B. C.). The idea of infinity has been combined with that of dimensions, e. g., infinity in one direction (एकतो अनंत), infinity in two directions (द्विधानंत), infinity in area (देश विस्तारानंत), infinity everywhere (सर्व विस्तारानंत) and infinity perpetual (शाश्वतानंत). It is mentioned that : अथवा पंच विद्या अनंत प्रज्ञप्ता: तद्यथा एकतो अनंतम् द्वि विधानन्तं, देश विस्तारानन्तं, सर्वविस्तारानन्तं, शाश्वतानन्तम्. While in Dhavalā and some other Jain philosphical texts, ten types of infinity have been described, e. g., nominal attributed, fluent, numerical, diamensionless, mono, bi, areal, spatial, phase and indestructible (everlasting).

In Kalpa-Sūtra and Navatattva (both works of 300 B. C.) infinity is described as a number as great as the number of sand grains on the brinks of all rivers on the earth or the drops of water in all the oceans. The Tilloypaṇṇatti, another important Jaina text, deals with infinity under mathematical disciplines. The Jaina concept of infinity in Mathematics can be explained in modern technology as, "If the law of variation of a magnitude x is such that it becomes and remains greater than any pre-assigned magnitude, however large, then x is said to become infinite and this concept is denoted by ∞.

The Jaina works on mathematics also deal with rules of operations with numbers, permutations and combinations, solutions of simultaneous equations, indeterminate equations of the first degree, laws of indices, arithmetical and geometrical progressions, rules for operation with infinity, mensuration formulae for different surfaces and solid bodies and many other topics.

In Chapter I on Introduction of this book, the author has mentioned

as remarked above that for the study of Cosmos and Karma theory, the Jaina scholars had developed mathematics very much. But it is unfortunate that in spite of some scattered attempts now by some research workers, like Raṅgāchārya, Smith, Kāpadiyā, Singh, Datta, Saraswati etc., most of the Jaina School mathematics is still unknown to the world, specially to the modern historians. In order to bring out this hidden treasure to light, the author has made a modest attempt. He describes in brief some facts about Gaṇitasāra-saṁgraha of Mahāvīrācārya (c. 850 A. D.) and also some other texts belonging to Karaṇānuyoga group of the Jainas. It is believed that the number concept based on the set theory was evolved at the hand of Georg Cantor about a hundred years ago but the author, on the basis of researches made by himself and other research workers, states that this concept originated in the Jaina School in the early Christian era and was symbolized to its perfection upto the time of the Jain Scholar Ṭoḍaramala of the 18th Century A. D. He further remarks that had the Karma system Theory of the Jainas in India with set theoretic approach reached Gauss, Fourier, Bolzano, Galois, Hamilton, Boole etc., the system theories of the biophysical world could have taken a new turn by now.

In Chapter 2, the author gives a synopsis of the world history of mathematics upto the time of Mahāvīrācārya (c. 850 A. D.). He also states, "In what part of the world and when, one thought that number and form could be as much useful as the language for any civilization, is not known. The present structure of mathematics has however been laid down on the basis of number and form. From the number system, were evolved arithmetic and algebra whereas the form study was responsible for the evolution of geometry. By the seventeenth century, the harmonious phases of these both led to the development of mathematical analysis." The author describes in brief the known works on mathematics by some ancient civilizations Sumerian, Babylonian, Egyptian, Chinese, Greek and Indian, the seats of ancient culture, as brought to light by several research workers. These works belong to the period, say, from 3000 B. C. or even earlier to the early centuries of the Christian era. A brief description of Indian mathematics from Āryabhaṭa I (5th Century A. D.) to the 16th Century A. D. is also given.

In Chapter 3, the author gives a nice and lucid description of the contents of the famous Jaina mathematical work 'Tiloyapaṇṇatti' by Yati vṛṣabha of 5th century A. D. This work belongs to the Karaṇānuyoga group of the Jaina Prakrit Texts, the other main books belonging to this group being Jambūdīva paṇṇatti saṁgaho, Lokavibhāga, Trilokasāra, Sūrya-prajñapti, Candra-prajñapti, Jambūdvīpa-prajñapti etc. This text gives a systematic treatment of basic mathematics as applied to the Jaina Karma

mathematical theory and describes the various units of simile measures for measuring distances and time (from minutest to the biggest) with the help of which the universe and all the fluents and their events are measured. It also gives the description of the 'Number measure.'

The minutest distance is the distance-space occupied by an ultimate particle (paramāṇu) and is called a pradeśa, that is, a space point, while the biggest distance is a rajju (or a jagaśrènī=7 rajjus) which is equal to asaṁkhya yojanas which in turn has been estimated to be equal to nearly 8 miles of the modern times. Similarly the smallest time, viz., "samaya" or "instant" has been defined as the time taken by a paramāṇu in moving from one space point to the next space point with minimum velocity; this unit of time is indivisible while the biggest practical unit of time is called 'acalātma' and is equal to $(84^{31} \times 10^{90})$ years.* In fact these texts also describe the system of infinite time measures. The two sets relating to distance and time are connected as follows** on the basis of cardinality :

$$\text{Log}_2 \text{ (Aṅgula)} = [(\text{Log}_2 \text{ (Palya)}]^2$$

where 'Aṅgula' means actually the number of 'Pradeśas' in one Utsédha-sūcyāṅgula (aṅgula) while 'Palya' means the number of 'Samayas' in one 'Palya' (actually palyopama).

This chapter also gives the description of the Number measure, including the concept of infinity. In fact the Jaina School needed a quantitative analysis of Karmic events and for it the concept of mathematical infinity was necessary. Twentyone types of numbers (including three types of saṁkhyāta, nine types of asaṁkhyāta and nine types of 'ananta, thus including eighteen types of infinities) were given, ranging from unity to the cardinal number of the indivisible-corresponding-sections of omniscience (all knowledge or Kevala-jñāna). Thus while the concept of infinity has been perfected by modern western mathematicians only recently, in India this had reached its climax even before the early centuries of the Christian Era.

Proceeding further to Chapter 4, the author throws further light on some salient features of mathematics discovered in another important Jaina work of 'Dhavalā' whose author Vīrasénācārya (8th century A. D.) was fully conversant with the place value system of notation, laws of indices, logarithm of a number to base 2 (called ardha cheda), general laws of logarithms, function of function-etc. Thus the principle of logarithm

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*According to the Śvetāmbara, texts, this unit is called Śīrṣa-Prahelikā and is equal to $(84^{28} \times 10^{140})$ years.

**As given in other relevant texts.

(called ardha-cheda at that time) and its rules of operation were known in India much earlier even before Napier (1550 A. D. to 1617 A. D.) and Burges (1552 A. D. to 1632 A. D.) discovered the logarithms. The author further shows that the examples used by Vīrasènācārya in his Dhavalā were based on his knowledge of fraction-manipulation, finite aud transfinite sets, geometric representation of algebraic equations and some processes extending to infinity etc. Vīrasènācārya has further quoted a new value of π, viz. $\frac{355}{113}$ (apart from 3, 10, $\frac{22}{7}$) from some earlier work, showing that this value of π (called later on as Chinese value of π) was also known in India perhaps earlier than in China.

In Chapter 5, the author gives a description of certain common topics of basic mathematics as contained in various ancient Jaina canonical works and discovered by himself and by Kāpadiyā, Datta, Singh Nemichandra, Saraswati, Gupta etc. Topics like Parikarma (fundamental operations), Vyavahāra (subjects of treatment), simple equations, quadratic equations, varga (square), ghana (cube), Varga-Varga, Vikalpa (abstraction or permutations and combinations), place value notation, numeral systems, sequences (specially of dyadic type) and progressions (arithmetical and geometrical), mensuration (dealing with circles, cylinders, cone, pyramids, prisms and their frustra, symbolism etc. have been dealt with.

In Chapter 6, the author compares in brief some ancient mathematical schools, the Egyptian, the Greek and the Chinese with the Jaina School and remarks that the Jaina School of Mathematics formed one of the important sources of transmission and influence.

In the end the author has given a comprehensive bibliography in three parts :

(a) Source material (b) References and Books (c) Research Articles, which can be useful to any research worker of ancient Jain School of Mathematics.

No doubt the value of the works like the present one is more historical than mathematical during these days of advanced mathematics but the contributions made by the Indian Scholars in general and the Jaina Scholars in particular, to the development of mathematics during the period from nearly 500 B. C. to 1200 A. D. or so is itself a great achievement, now acknowledged even by western scholars, and deserve labour and sincere efforts in bringing out the same to the notice of the modern world. The present work by Prof. L. C. Jain is a modest attempt in this direction. He has got a keen interest in exploring the secrets of ancient mathematics developed in India and has a vast experience of more than two decades in this field. He

has himself contributed a large number of expository, survey and research articles on ancient Indian mathematics, specially on Jaina School of mathematics. With this experience to his credit, he has very ably achieved what he wanted to achieve in this book on 'Basic Mathematics, Vol. I, of Exact Sciences from Jaina Sources.'

It is hoped that this work would prove to be an important source of reference for any future worker in this field.

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THE JAINA SCHOOL OF EXACT SCIENCES

PART I

BASIC MATHEMATICS

CHAPTER 1.

INTRODUCTION

"Mathematics is the science of the functional laws and transformations which enable us to covert figured extension and rated motion into number". —**Howison, G. H.**[1]

According to J. J. Sylvester, there are three ruling ideas, three to say, spheres of thought, which pervade the whole body of mathematical science, to some one or other of which, or to two or all three of them combined, every mathematical truth admits of being referred; these are the three cardinal notions, of Number, Space ond Order. Arithmetic has for its object the properties of number in the abstract. In algebra, viewed as a science of operations, order is the predominant idea. The business of geometry is with the evolution of the properties of space, or of bodies viewed as existing in Space[2].

(a) Object :

To a human intellect two worlds have been manifest for study of the cosmos. One has been the micro world and the other macro one. The observables could be calculated for their motion and transformations through postulated non-observables.

With this object, in India, the Jaina scientific thought contributed to Jaina astronomy as well as to Jaina Karma theory with mathematical approach. One of the works on basic mathematics became world known,

1. Journal of Speculative Philosophy, Vol. 5, p. 164.
2. Collected Mathematical Papers, Vol. 2, p. 5.

as Gaṇitasārasaṁgraha[3] of Mahāvirācārya (+9th century), others being known as astronomical treatises, the Sūryaprajñapti and the Candraprajñapti[4].

Mahāvirācārya expressed his obligation to the tradition of previous mathematicians, "With the help of the accomplished holy sages, who are worthy to be worshipped by the lords of the world, and of their disciples and disciples' disciples, who constitute the well-known jointed saries of preceptors, I glean from the great ocean of the knowledge of numbers a little of its essence, in the manner in which gems are picked up from the sea, gold is from the stony rock and the pearl from the oyster shell; and give out according to the power of my intelligence, the Sāra-Saṁgraha, a small work on arithmetic, which is (however) not small in value."[5]

As one of the best mathematicians of his time, he gives his appreciation of mathematics in the following words, "In all transactions which relate to worldly, Vedic or other similar religious affairs, calculation is of use. In the science of love, in the science of economics, in music and in drama, in the art of cooking, in medicine, in architecture, in prosody, in poetics and poetry, in logic and grammar and such other things, and in relation to all that constitutes the peculiar value of the arts, the science of calculation is held in high esteem. In relation to the movement of the sun and other heavenly bodies, in connection with eclipses and conjunction of planets, and in connection with the tripraśna (direction, position and time) and the course of the moon – indeed in all these it is utilized. The number, the diameter and the perimeter of islands, oceans and mountains; the extensive dimensions of the rows of habitations and halls belonging to the inhabitants of the world, of the interspace between the worlds, of the world of light, of world of gods and of the dwellers in hell, and other miscellaneous measurements of all sorts—all these are made out by the help of gaṇita. The configuration of living beings therein, the length of their lives, their eight attributes, and other similar things; their progress and other such things, their staying together, etc.—all these are dependent upon gaṇita (for their due comprehension). What is the good of saying much ? Whatever there is in all the three worlds, which are possessed of moving and non-

3. Edited in +1912, with English Translation by Raṅgācārya, Madras. Cf. also GSS. edited & translated by L. C. Jain, Sholapur, 1963.

4. These constitute as two of the twelve upāṅgas, whose tradition is traced back to—3rd century at Patliputra. Cf. bibliography also.

5. Cf. pp. 17-19, (op. cit.)

moving beings, cannot exist as apart from gaṇita (measurement and calculation)[6]."

Mahāvīrācārya further points out, "Thus the terminology is stated briefly by the great sages. What still remains to be said should be learnt in detail from the Āgamas[7]".

(b) Texts on Karaṇānuyoga group :

The above description clearly leads one to the Āgamas (on Karma theory in particular) wherein lie the deep secrets of periodic events in nature involving as many as ten types of infinities.[8] The main texts of the Karaṇānuyoga group, in which either mathematics has been developed as post-universal study of measures or applied in form of results through analysis and comparability, are a few one. They are the Kasāyapāhuḍa, the Ṣaṭkhaṇḍāgama, the Tiloyapaṇṇatti, as well as their extracts in form of the Kṣapaṇāsāra (including Labdhisāra), the Gommaṭasāra, and the Trilokasāra. Their Dhavalā, Jaidhavalā, Mahādhavalā, Jīvatattva pradīpikā and Samyakjñānacandrikā commentaries deserve special attention of mathematicians.[9]

Research on Jaina School of Mathematics has been very active after Raṅgācārya and Smith[10] who found rich and resourceful guidelines from Mahāvīrācāryaś Gaṇita Sāra Saṁgraha, after + 1912. Since then the Dhavalā and the Tiloyapaṇṇatti texts inspired Singh to write in more details after an introductory article by Datta, at the instance of Dr. H. L. Jain who was engaged in the translations of the Prakrit Ṣaṭkhaṇḍāgama and other texts for a period of no less than thirty years into Hindi. After contributing several articles on Jaina Mathematics, Datta, had still felt diffident in manipulating his articles,[11] and remarked, "The writer who has only recently began collecting materials for a full and comprehensive account of the contribution

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6. Cf. ibid. pp. 9-10.
7. Cf. ibid. p. 70.
8. Nominal, attributed, fluent, numerical, dimensionless, mono, bi, areal, spatial, phase, indestructible (everlasting) infinities, Cf. Dhavalā, III, pp. 11-16.
9. Cf. bibliography.
10. Cf. bibliography of research papers on Jaina School of Exact Sciences.
11. The Jaina School of Mathematics, op. cit., B.C.M.S., 21 (1929), 115-145.

by the Jaina scholars to the development of Hindu Mathematics, will have to refrain from further prosecuting his project now at this preliminary stage of the investigation. So he wishes to keep in print a brief account of the results obtained by his labour in the hope that it will probably save the future and more successful researcher at least of some amount of his labour. Moreover, even within this short span of time, there have been discovered certain mathematical results which are not only highly interesting but are also considered very important for the history of Hindu mathematics."[12]

When the Dhavalā texts III was sent to Dr. A. N. Singh, by Dr. H. L. Jain for writing an article on the mathematics embedded therein, he was not prepared to take it up for the reason that he did not think himself competent then, because of the technical terminology involved in the texts. However, by long persuasion, as Dr. H. L. Jain used to relate, he contributed three articles.[13] He expresses in, "Mathematics of Dhavalā", the following impression, "We have been accustomed to think that amongst the vast population of India only the Hindus studied mathematics and were interested in the subject, and that the other sections of the population of India, e. g. the Buddhists and the Jainas, did not pay much attention to it. This view has been held by scholars because mathematical works written by Buddhists or Jaina mathematicians had been unknown until quite recently. A study of the Jaina cononical works, however, reveals that mathematics was held in high esteem by the Jainas. In fact the knowledge of mathematics and astronomy was considered to be one of the principal accomplishments of the Jaina asceties[1] (Cf. Bhagwati-sūtra with the commentary of Abhayadeva Sūri, edited by Āgamodaya samiti of Mehesana, 1919, Sūtra 90; English Translation by Jacobi of the Uttarādhyayana-sūtra, Oxford, 1895, ch. 7, 8, 38)".[14]

Dr. Singh has further noted, "Although the names of several Jaina mathematicians are known, their works have been lost. The earliest among them is Bhadrabāhu who died in 278 B. C. He is known to be the author of two astronomical works : (i) a commentary on the Sūryaprajñpati and (ii) an original work called the Bhadrabāhavī Saṃhita. He is mentioned by Malayagiri (c. 1150) in his commentary on the Suryaprajñapti, and has been quoted by Bhaṭṭotpala (966)[1] [Bṛhat Saṃhitā, ed. by S. Dvivedi, Benaras, 1895, p. 226.] Another Jaina astronomer of the name of Siddhasena has been quoted by Varāhamihira (505) and Bhaṭṭotpla. Mathematical

12. Cf. ibid.
13. Cf. bibliography.
14. Cf. ibid., Dhavalā, vol. 3, Amaraoti, 1942.pp. i–xxi.

quotations in Ardha-māgadhī and Prakrit are met with in several works. The Dhavalā contains a large number of such quotations. These quotations will be considered at their proper places, but it must be noted here that they prove beyond doubt the existence of mathematical works written by Jaina Scholars which are now lost[2]."[15]

Śilāṅka has commented on Sūtrakṛtāṅga Sūtra, Smayadhyayana, Anuyogadvāra, verse 28, quotes three rules regarding permutations and combinations, which belong to some Jaina mathematical work.

Bhāskara I has also quoted a number of passages in Prākṛta verses and seem to have been taken from Jaina sources.[16] Mathematical Works of Jaina scholars under the names, Kṣetra-Samāsa and Karaṇa-bhāvanā, are not available. Similarly Yativṛṣabha quoted verses from Aggāyaṇī, Diṭṭhivāda, Parikamma, Mūlāyāra, Loyaviṇicchaya, Loyavibhāga and Logāiṇī, which are not traceable now.[17]

(c) Recognition :

Singh recognised that the knowledge of Jaina mathematics which is of an extremely fragmentary character is gleaned from a few non-mathematical works as Sthānāṅga-sūtra, Tattvārthādhigama–sūtra–bhāṣya of Umāsvāti, Sūryaprajñapti, Anuyogadvāra-sūtra, Trilokaprajñapti, Triloksāra, Jambūdvīprajñapti, Lokavibhāga, Bhagavati-sūtra, Kalpasūtra, Candraprajñapti, Jyotiṣakaraṇḍaka, etc., to which now be added the Dhavalā.[18] Regarding the importance of the Dhavalā, Singh remarks, "The Dhavalā was written by Vīrasena in the beginning of the ninth century. Vīrasena was a philosopher and religious divine. He certainly was not a mathematician. The mathematical material contained in the Dhavalā may therefore be attributed to previous writers, especially to the previous commentators of whom five have been mentioned by Indranandi in the Śrutāvatāra. These commentators were Kundakunda, Shamakunda, Tumbulura, Samantabhadra and Bappadeva, of whom the first flourished about 200 A. D. and the last about 600 A. D. Most of the mathematical material in the Dhavalā may therefore be taken to belong to the period 200 to 600 A. D. Thus the Dhavalā becomes a work of first rate

15. Cf. ibid.

16. Shukla, K. S., Hindu Mathematics in the seventh century as found in Bhāskara–I's commentary on Āryabhaṭīya, Gaṇita, 22.1, 115–130; 22.1, 61–68; 23.1, 57–79; 41–50.

17. Tiloyapaṇṇatti, part II., intro. 11–12.

18. Cf. bibliography.

importance to the historians of Indian Mathematics—the period preceding the fifth century A. D.[19] The study of the analytical methods for comparing infinities led him to opine, "As already pointed out, the method of one-to-one correspondence has proved to be the most powerful tool for the study of infinite cardinals, and the discovery and first use of the principle must be ascribed to the Jainas.[20]"

Datta and other scholars[21] did not have before them detailed commentaries of the Gommaṭasāra, Kṣapaṇāsāra texts on which two separate chapters on artha saṃdṛṣṭi (symbolic norms) were compiled by Ṭoḍaramala of Jaipur[22] (c + 1767), serving as guides to the learners. His predecessors, Keśavavarṇī (c. + 1360) and Nemichandra (c. + 1552), a disciple of Jñānabhūṣaṇa, had paved the way through Kannaḍa and Sanskrit commentaries wherein one could find two types of mathematical measures : universal and post-universal. These commentators had before them the Dhavalā texts as well as the Tiloyapaṇṇatti, alongwith the summary texts of Nemichandra Sidhāntacakravartī (c.+11th century). The texts are on the mathematical theory of Karmic system.

(d) Events :

It is strange to observe, how the western and the eastern parts of the world were deeply absorbed in mathematical manoeuvre. The mathematical world remodelled by Newton was facing the well known crisis in calculus. It was gradually progressing from intuition to absolute rigour. Euler (+1750) made profound changes in analysis and Lagrange (+ 1780) contributed to the theory of numbers, analysis and elliptic functions. It was a dead mathematical world and the number concept based on a set theory was still to evolve at the hand of Georg Cantor after a century, which in the Jaina School originated in the early Christian era and symbolized to its perfection upto the time of Ṭoḍaramala. Quantum theory was to appear still later, alongwith applications of set theory and functional analysis. Had the Karma system theory of India with its set theoretic approach reached Gauss (+ 1800), Fourier (+ 1810), Bolzano (+ 1820), Galois (+ 1830), Hamilton (+ 1850) and Boole (+ 1850), the systems theories of the biophysical world could have taken a new turn by now.

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19. Cf. Mathematics of Dhavalā, op. cit.
20. Cf. ibid.
21. Vid, the works of the more research workers Kapadia, Misra, Roy, Jain, Shastri, Saraswati, Volodarsky, Gupta, Vijayaraghavan, Lishk, Sharma, Agrawal, M. Kumar, Sikdar, Zaveri, Bose, Sen, Subbarayappa, Bag and so on, referred in bibliography.
22. Cf. Mathematical contribution, op. cit. bibliography.

Writing on the mathematics of Trilokaprajñapti, Sarasvati[23] writes, "The Trilokaprajñapti or Tiloyapaṇṇatti, a Karaṇānuyoga text of the Jainas, deals with Jainā cosmography, expiating on the order, number and measurements of the dvipas, and samudras, the heavens, the hellish pits or bilas, the mountains and the rivers of the universe. Since all these are given a regular pattern and a geometric shape, mathematics becomes an indispensable tool for this semi-religious discussion. And if the unwillingness of the Jainas to change their fantastic geography and astronomy in the light of the more advanced knowledge of the siddhāntas, is a pointer to their strong conservatism, the methematical knowledge contained in such Jainā works as the Trilokaprajñapti, the Jambūdvīpa-prajñapti and the Bṛhatkṣetrasamāsa must be considered very ancient. In fact, it is highly probable that even the present version of the TP is anterior to the Mathematician Mahāvīrācārya and even Śrīdharācārya." She further opines, "According to Sri Nemicandra Jain,[3] (तिलोय-पण्णत्ती की प्रशस्ति Jainā Antiquary, vol. X, No. 11), the mathematical rules in the TP fall into different periods beginning with the Vedāṅga Jyotiṣa period and ending with that of Bhaskar II. I doubt if we are justified in bringing down the date of the TP on the basis of this classification; for, in India where Mathematics is only a handmaid to astronomy, cosmography and practical life, it is rather hazardous to assign a particular level of Mathematical knowledge to a particular period."[24]

According to Saraswathi, the non-preservation of records and the exclusively religious design of the preserved texts make it difficult to trace the growth of scientific ideas in acient India. The early Jaina canonical works and Pingala's Chandaḥsūtras belonging to this period are familiar with series mathematics, combinations, circle geometry and computations of the areas of simple geometrical figures.[25] Unaware of the Ṭoḍaramal's works, she concludes, "After the middle of the sixteenth century no new ground seems to have been covered. Even in the preceding centuries the torch was kept burning only in India's isolated corner strip, Kerala. This also was put out by the wind of European incursions into the Malabar coast. Dazed by fire-arms and dazzled by the enterprise and material advancement of the intruders, people began to look down upon native scholarship and achieve-

23. The Journal of the G. J. R. I., Allahabad, vol. 18. Nov. 1961, Aug. 1962 pp. 27–52.

24. Cf. ibid.

25. "Development of Mathematical Ideas", I. J. H. S., vol. 4, nos. 1 and 2, 1969. pp. 59–78.

ments and the pursuit of mathematics died a natural death alongwith the other arts and sciences of native origin,"[26]

(e) Questions :

Devid Eugene Smith[27], in his introduction to Gaṇita Sāra Saṃgraha of Mahāvīrācārya, raises leading questions in connection with the work, "In considering the work, the reader naturally repeats to himself the great questions that are so often raised :—How much of this Hindu treatment is original ? What evidences are there here of Greek influence ? What relation was there between the great mathematical centres of India ? What is the distinctive feature, if any, of the Hindu algebraic theory ? Such questions are not new. Davis and Strachey, Colebrooke and Taylor, all raised similar ones a century ago, and they are by no means satisfactorily answered even yet. Nevertheless, we are making good progress towards their satisfactory solution in the not too distant future. The past century has seen several Chinese and Japanese mathematical works made more or less familiar to the West; and the more important Arab treatises are now quite satisfactorily known. Various editions of Bhāskara have appeared in India; and in general the great treatises of the Orient have begun to be subjected to critical study. It would be strange, therefore, if we were not in a position to weigh up, with more certainty than before, the claims of the Hindu algebra. Certainly the persevering work of Professor Raṅgācārya has made this more possible than ever before."

The detailed commentaries of Ṣaṭkhaṇḍāgama and Kaṣāyaprābhṛta down to the eighteenth century were not exposed to Smith, nor known to him, still he could give a perfect analysis of the differentiation between the Greek and Indian contributions in the following words, "As to the relation between the East and the West, we should not be in a position to say rather definitely that there is no evidence of any considerable influence of Greek algebra upon that of India. The two subjects were radically different. It is true that Diophantus lived about two centuries before the first Āryabhaṭa, that the paths of trade were open from the West to the East, and that the itinerant scholar undoubtedly carried learning from place to place. But the spirit of Diophantus, showing itself in a dawning symbolism and in a peculiar type of equation, is not seen at all in the works of the East. None of his problems, not a trace of his symbolism, and not a bit of his phraseology appear in the

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26. Cf. ibid.

27. Gaṇitasāra Saṃgraha of Mahāvīrācārya, ed. & trans. by L. C. Jain, Sholapur, 1962, 72–77, Appendix.

works of any Indian writer on algebra. On the contrary, the Hindu works have a style and a range of topics peculiarly their own. Their problems lack the cold, clear, geometric precision of the West; they are clothed in that poetic language which distinguishes the East, and they relate to subjects that find no place in the scientific books of the Greeks. With perhaps the single exception of Metrodorus, it is only when we come to the puzzle problems doubtfully attributed to Alcuin that we find anything in the West which resembles, even in a slight degree, the work of Alcuin's Indian contemporary, the author of this treatise."[28]

Thus Smith records that Mahāvīrācārya's work gave further evidence of the fact that Oriental mathematics lacks the cold logic, the consecutive arrangement, and the abstract character of Greek mathematics, but that it possesses a richness of imagination, an interest in problem setting, and poetry, all of which are lacking in the treatises of the West, although abounding in the works of China and Japan.

Same more vital questions and doubts are raised by him, which again raise the indication about motivation and origination of the algebraic achievements, "When we come to the question of the relation between the different sections of the East, however, we meet with more difficulty. What were the relations, for example, between the school of Pāṭaliputra, where Āryabhaṭa wrote, and that of Ujjain, where both Brahmgupta and Bhāskara lived and taught ? And what was the relation of these to the school down in South India, which produced this notable treatise of Mahāvīrācārya ? And a still more interesting question is, what can we say of the influence exerted on China by Hindu scholars, or vice versa ? When we find one set of early inscriptions, those at Nānā Ghat, using the first three Chinese numerals, and another of about the same period using the later forms of Mesopotamia, we feel that both China and the West may have influenced Hindu science. When on the other hand, we consider the problems of the great trio of Chinese algebraists of the thirteenth century, Ch'in Chiushang, Li Yeh, and Chu Shih-chieh, we feel that Hindu algebra must have had no small influence upon the North of Asia, although it must be said that in point of theory the Chinese of that period naturally surpassed the earlier writers of India."[29]

It may also be noted that the authors of, "A concise history of science in India"[30] seem to have remained unware of the *Saṭkhaṇḍāgama*, Kasāyapāhuḍa,

28. Cf. ibid, opp. p.-74.
29. Cf. Ibid., pp. 74-75.
30. Cf. Bose, D.M.; Sen, S.N. and Subbarayappa, B.V., op. cit, New Delhi 1971.

their monumental Dhavalā commentaries; Tiloyapaṇṇatti, Trilokasāra; and the Gommaṭasāra, Labdhisāra as well as their mathematical and symbolic commentaries. Even the research papers of A. N. Singh and Saraswathi were not before them, however, their remarks are worthy of attention, "Despite great interest attached to mathematics by the Jainas of ancient India very little of their mathematical effort has unfortunately survived to this day...... That a good body of mathematical literature must have existed at one time seems indicated in Mahāvīra's own statement in which he described himself as a mere compiler from the great ocean of the knowledge of mathematics from which long lines of holy sages had skilfully gathered many precious mathematical gems..... These subjects and the various technical terms used by the Jainas passed later on into the mathematical works of scholars irrespective of their religious beliefs and adherences. It is thus quite reasonable to believe that in the period intervening the literature of the Brāhmaṇas and the Sūtras of the various Vedic schools and the period of specialization and Siddhāntic astronomy from about the fourth or fifth century A. D. the Jaina mathematicians played a significant role. The disappearance of their works might be due to (a) supersession of their simple processes and methods by better and more sophisticated ones due to Āryabhaṭa, Brahmagupta, Mahāvīra, Śrīpati, Śrīdhara, Bhāskara and others and (b) progressive deterioration of the culture of mathematics in their religious order."[31]

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31. Cf. ibid., pp. 157-158.

CHAPTER II

A SYNOPSIS OF WORLD HISTORY OF MATHEMATICS UP TO MAHĀVĪRĀCĀRYA

"Mathematics is a self-contained microcosm, but it also has the potentiality of mirroring and modelling all the processes of thought and perhaps all of science. It has always had, and continues to an ever increasing degree to have, great usefulness. One could even go so far as to say that mathematics was necessary for man's conquest of nature and for the development of the human race through the shaping of its modes of thinking."[32]

—Mark Kac and Stanislaw Marcin Ulam

It is unknown, in what part of the world, and when one thought that number and form could be as much useful as the language, for civilization. The present structure of mathematics has been laid down on the basis of number and form. From the number system were evolved arithmetic and algebra, whereas the form study was responsible for evolution of geometry. By the seventeenth century, the harmonious phases of both led to the development of mathematical analysis.

Even hundreds of years before Christ, the countries and lands recorded their contemporary knowledge through material which could survive upto the present day. Out of these, the ancient civilizations of Sumer and Babylon, Egypt, India as well as China are famous in the whole world.

(a) Sumer and Babylon :

According to Neugebauer,[33] for the old Babylonian texts (Hammurapi dynasty : c.-1800 to – 1600), prehistory cannot be given regarding Sumerian development. He it of the opinion that Babylonian mathematics was not gradually brought to its high level, but as a rapid growth, based on the preceding development of the sexagesimal place value system

32. Britannica Perspectives, vol. I, 1768–1968, 1968, London.

33. "The Exact Sciences in Antiquity", Providence, 1957, pp. 29 et. seq.

whose rudimentary forms have already been attested in numerous economic texts (table and problem) from the earliest phases of written documents (about 500,000 cuneiform tablets) found in the ruins of Mesopotamia and Nippur. The table texts on the tablets have vocabularies basic to scribal instructions and indispensable for the mastery of the secrets of cuneiform writing in Akkadian and in Sumerian. The problem texts mixed with those of weights & measures (c.–2500) were needed for routine matters. This is the western region of the Tigris and Euphretes rivers where civilization cropped up since –5700. The people preserved their learning in baked earth found in forms of circular, cylindrical and prismatic tablets. Agriculture was the base of their civilization requiring a calendar, year beginning from vernal equinox. The Babylonians learnt from the Sumerians the measures based on sexagesimal, which also contained the partial decimal system; ten, perhaps from fingers, and six, perhaps from simplicity of manipulating fractions.

From – 2000, their tables of arithmetic contained not only multiplication but also squares, cubes and square-roots. The tables of n^3+n^2 were needed for n from 1 to 30, proving their tendency towards functionality, with observation and proof. Equations were partially solved in forms of $x^3+ax^2+b=0$, through the tables. They could also solve linear equations in ten unknowns. They could also find out areas of rectangles, right-angled triangles etc., and the value of π was denominated as 3. For irrigation purposes the inhabitants could solve the problems for volumes of right circular cylinders and prisms. They knew that the angle in the segment of semicircle is right angle, that in a right angle the hypotenuse square is equal to the sum of squares of its base and perpendicular and that the corresponding sides of right angled triangles are proportional. Arithmetical concept "ratio" had a special term. The value $\pi \approx 3\frac{1}{8}$ does not appear to be attested in the preserved literature of this period. After this no more progress is available here as it seems.

(b) Egypt :

For Neugebaur finds Egyptian[34] ancient the most pleasant, without excessive development of Greek heroism, spared from struggles, full of pyramids and agricultural culture. This is the gifted plain of the Nile, with records of archaeology from–4000 to–2781. A calendar (c.–4241) represents 12 months of 30 days each, to be added by five days for completion of a year. The records could be preserved in the desert's climate through papyri. The Egyptians had to wait with their old

34. Cf. ibid., pp. 71 et. seq.

mathematics without any application to navigation or astronomy or economics till the Hellenistic period (Alexander the Great).

The whole procedure of Egyptian mathematics is additive, and the development of computation with the help of fractions. From the papyri, it is known that c.–3500, numbers ranging to crores were mentioned. The heieroglyphics script depicts that 1,20,000 human beings, 4,00,000 oxen and 14,22,000 goats were imprisoned. After computation they also tried to follow a decimal system which was without place-value.

The arithmetic of –1650 contains multiplication and division. In Rhind papyrus $\frac{2}{97}$ is expressed as $\frac{1}{56}+\frac{1}{679}+\frac{1}{776}$. From a problem of c.–2000, they perhaps could solve an equation of the type $x^2+y^2=100$, $y=\frac{3}{4}x$ in algebra, by making use of the rule of false position which was in vogue up to the + 15th century.

The Egyptians had the knowledge of proportion as well, which is the basis of mathematical analysis. The value of π (ratio of circumference to diameter) was found to be $\frac{256}{81}$ or 3·16. There is no evidence of the use of Pythagorean theorem, yet the proportion in the rope stretchers was 5 : 4 : 3. The practical measures were in use before –3000. The water gauges all around for 700 miles were in the same level. The volume of cylinders as well as area of triangle were known correctly. From the Moscow papyrus it appears that even c.–1850, some unknown mathematician had given for the volume of a frustrum of a pyramid in the form : volume$=\frac{1}{3}h(a^2+ab+b^2)$, where a, b are measures of the base sides of upper and lower planes, h being the height. The method of exhaustion has been used here.

Thus the part played by Egyptian mathematics is probably best described as a retarding force on the procedures making use of numbers. They could not go beyond the above achievement, as it appears.

(c) China :

A very exhaustive work by Needham and Ling[35] elucidates the ancient Chinese mathematics and its further development, which also comprises the study of the literature on history of Chinese mathematics by Cantor, Loria, Cajori, Smith, Karpinski, Biot, Biernatzki, Wylie, Sèdillot, Archibald, van der Waerden, Mikami Yoshio and Yushkevitch and so on. According to Needham and Ling, "One must realize that there is an absolutely continuous tradition between the Chinese written language as found on the oracles bones of the 14th century and the language as written and spoken today.

35. "Science and Civilisation in China", Cambridge, Vol. III, 1959.

Analogy with Sumerian or ancient Egyptian is therefore not valid; Hebrew doubtfully competes."[36]

Strong reasons have been given that influence of Mesopotamian culture was only very limited in China. Apart from a few traditional cycles, there is little sign of sexagesimal arithmetic, and no special symbol for the fraction $\frac{2}{3}$. This fraction was considered very important in Mesopotamia.

A numerical notation of only nine figures combined with place-value components have been found in China as early as −14th century. In the primitive form, the zero, the black space on the counting board appears in −4th century.

The extraction of square and cube roots was in a developed stage in −1st century. Rule of three is found in Han Chiu Chang, earlier than any Indian Sanskrit text. Negative numbers appear in −1st century, the Pythagorean theorem in +3rd century commentary of Chou Pei, geometrical survey material in +3rd century commentary of Liu Hui. The fundamental identity of algebraic and geometrical relations was appreciated by Chinese throughout the previous millennium. Indeterminate analysis was taken up in +4th century in Sun Tzu.

It is quite strange to see that Euclid's geometry reaches China by +1275, so also trigonometric methods seem to have been in use in +1270.

Thus Mikami opines that the greatest deficiency in old Chinese mathematical thought was the absence of the idea of rigorous proof. He correlates this with the failure of formal logic to develop in China. Chinese mathematicians never spontaneously invented any symbolic way of writing formulae. Land mensuration, survey, granary dimensions, making of dykes and canals, taxation, rate of exchange, alongwith calendrical calculations were considered important. There was practically nothing of mathematics "for the sake of mathematics"[37].

(d) Greece :

From -600, Greece is world famous for its unparalleled progress in geometry and conics since −650. Thales and Pythagoras based mathematics on logic and illustrated the natural events through arithmetic. Yet the

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36. Cf. ibid. p. xlii (author's note).

37. Cf. ibid., pp. 151–153.

Euclid's "Elements" (c.–300) and Piolemy's "Almagest" (c. + 150) could reduce all their predecessors to objects of mere "historical interest"[38].

It is natural to imagine that the progress made in Greece since Pythagoras, had something to do with the events and knowledge in the Eastern countries. His first great discovery was the dependence of musical intervals on arithmetical proportions of lengths of strings in equal tension. Everyone is acquainted with the theorem of Pythgoras, which led to prove the irrationality of $\sqrt{2}$. The Pythagoreans found the linear solutions of quadratic equations with real roots, laws of proportions, showed the structure of five regular solids and gave similar figures of equal areas. Their figurate numbers became very useful in science. Numerology is said to have started from the Pythagorean era.

Neugébauer remarks that the theory of irrational quantities and the related theory af integration are of purely Greek origin, but the "geometric algebra" applied is of Mesopotamian origin.

Zeno of Elea (c. –495 ? –435 ?) produced four paradoxes of infinities. Eudous (– 408 to – 355) proved the Egyptian volume-formulas through law of proportion and founded the system of real numbers in axiomatic style. He is said to have visited the Eastern countries.

Euclid (– 365 to – 275) proved many theorems based on arithmetical division. He based geometry on logic and brought arithmetica to order.

Archimedes (– 287 to – 212) founded the mathemetical sciences of statics as well as hydrostatics, approximated integral calculus, and applied roughly the differential calculus through an equiangular spiral. He applied the method of exhaustion for finding out volumes of revolution, including hyperboloid. He was also the first to find out the sum of an infinite series.

Hipparchus (c. – 150) illustrated the orbits and velocities of planets through geometry. Heron (+ 200) gave a formula for finding out the area of any triangle in terms of its sides alone. Pappus (+250) studied the ellipsoid and other conic sections, discovered a theorem of projective geometry and found theorems for finding out volumes of solids. During this period, Diophantus solved linear equations with two and three unknowns.

Neugebauer summarizes the above earlier achievements as follows, "We know today that all the factual mathematical knowledge which is

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38. Cf. Neugebauer, op. cit., p. 145.

ascribed to the early Greek philosophers was known many centuries before, though without the accompanying evidence of any formal method which the mathematicians of the fourth century would have called a proof......It seems to me characteristic, however, that Archytas of Tarentum could make the statement that not geometry but arithmetic alone could provide satisfactory proofs."[39]

(e) India :

The team work of Datta and Singh[40] as well as that of Bose, Sen and Subbarayappa[41] have brought to light several main contributions of the Indians to mathematics since –3000. In their recent contributions, Yushkevich. Shukla, Gupta, Kupanna-Shastri, Bag, Saraswathi, Jha, Volodarsky, Pingri, Sharma and Lishk, Sen, Sharma, Majumdar, Ansari, Rambehari and Jain, Kulkarni and others have been continuously contributing articles and research papers on Indian ancient and medieval mathematics in the Indian Journal of History of Science and other Journals. The publication of works by Brahmgupta, Āryabhaṭa, Bhaskara I, Śridhara, Nārāyana, Nīlakaṇṭha, Parameśvara, then philosophical series of texts as Ṣaṭkhaṇḍāgama, Tiloyapaṇṇatti and Kasāyapāhuḍa alongwith their translations into contemporary language have added enormously to the spade work of Datta and Singh.

It is now well recognized that alongwith Assyria, Babylon and Egypt, India was one of the earliest seat of culture. The study of excavations of ancient settlements of Mohenjodaro and Harappa round about the Indus civilization sites had led the scholars to accept that the inhabitants apart from having mathematical and scientific knowledge, knew the fundamentals of construction, metallurgy, shipbuilding and navigation. India also played a great part in linking this knowledge with the East and the West in far off lands.

Since 1925, the excavations at the sites of Indus Civilization, neither texts concerned with mathematics have come to light, nor the pictograph inscriptions, seals and stone weights could be deciphered so far. Regarding arithmetic, in all probability, decimal numbering can be seen from a ruler

39. Cf. ibid., p. 148.

40. History of Hindu Mathematics, Pts I & II, Bombay, 1962.

41. Op. cit. Cf. also Bag, A. K., Symbol for Zero in Mathematical Notation in India, Boletin de la Academia Nac. de Ciencias, Córdoba, T. 48, 1970, 247-254.

in which points inserted 6.7 mm apart of which nine points remained preserved. Numbers were commonly marked with notch strokes, detected upto 9 except 8. However, some horizontal and vertical strokes survive in later Indian number system of the Kharoṣṭī and Brahmi.

Stone weights[42] of polished shale indicate the unit of weight as 0.8565 gm and its multiples of 10 and 100 as well as in doubling. with following correlation : $\frac{1}{8}$, 1, 2, 4, 8, 16, 32, 64, 160, 200, 320, 640, 1600, 3200, 6400, 8000, 12800. The rectangular bricks of 1 : 2 : 4 and 1 : 3 : 9 sizes, and wedge-shaped or of other forms are found in houses or wells.

Significance of Indus scale is that 25 Indus inches equal 33 inches which again equal the yard prevalent throughout north India in Akbar's period. Recently Kulkarni[43] his conjectured that knowledge of mensuration & geometry seems to have developed to a high degree of accuracy. The inhabitants might be knowing the approximate value of π, the theorem of Pythagoras & the construction to draw a rational right angled triangle, alongwith properties of a circle.

According to Voladarsky the highest artistic achievement is represented by the amulet seals, made of steatite, in square and rectangular forms. The seals consisted of syllabii signs with ideograms and perhaps with determinants. The deciphering of these texts is still awaited to add to the knowledge of the relations of Indian achievements in mathematics with the contemporary civilizations of Egypt and Babylon.

Mathematical knowledge from records is available in the various sections of Vedic literature : The Saṃhitas, the Brahmaṇas, and the Vedāṅgas. The Kalpasūtras (ritual) and the Jyotiṣa (astronomy) are concerned with constructions of sacrificial altars and with calendrical purposes. Of great importance are the Śulb-sūtrās which concern with rules of measurements with the help of a cord of various linear, spatial or three-dimensional figures. They represent Brāhmaṇa geometry or mensuration. These appear to be compiled between – 600 to – 300.

During the Vedic era, the Indians had an extensive knowledge of mathematics, of a developed system for designating numbers, of summations

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42. Cf. A. I. Volodarsky, Mathematics in Ancient India, XIV Inter. Cong. of Hist. of Sc., Tokyo, Aug. 1974, pp. 356–359.

43. "Geometry as known to the People of Indus Civilization", I. J. H. S., vol. 13, no. 2, nov. 1978, 117–124.

of a few types of arithmetical and geometrical series, and of basic arithmetic operations with the whole and fractional numbers. They also knew how to transform plane figures, how to find areas and volumes. Some methods for solution of linear and quadratic equations of the first and second degrees, elements of combinatorial analysis and concepts of irrational values were also discovered by them.

There appears to be a gap of about 1000 years between the Vedas and Āryabhaṭa I. This is the darkest period of history of Indian mathematics as well as sciences when the Jainas and the Buddhists had sway over mathematical and scientific literature, gradually either swept out or utilized for further development by Āryabhaṭa I (+5th century), Brahmagupta (+6th century), Bhāskara I (+6th+7th centuries), Mahāvīra, Śrīdhara (+9th+10th centuries), Āryabhaṭa II (+10th century), Śrīpati (+10th+11th centuries), Bhāskara II (+12th century), Nārāyaṇa (+14th century), and Nīlakaṇṭha (+15th, +16th century). However, it may be emphasized that the aim of the Jaina scholars had been to expound their mathematical Karmic theory for which they distinctly developed a mathematics of post-universal measures in which numerate, innumerate, infinite as well as simile-set theoretic measures and norms were predominant. They needed indispensably some types of symbolic mathematics which could expose their philosophic details.

It is still open to debate as to who discovered the decimal place-value numeration, but it is certain that it might have been invented out of a great necessity of writing of very large numbers which were required invairably by the Jainas in exposing the various types of sets of souls in different rummage as well as control stations, in relating various Karmic periods and structures as well as categories and factors.

Āryabhaṭa I had before him, the astronomical knowledge of the Pauliśa, Romaka, Vāśiṣṭha, Saura and Paitāmaha schools, apart from that of the Jaina school, a conservative tradition carried upto the present century in the age-old form. He made use of the concepts and procedure of Kalpa, Meru, numeration system as well as the Greek's methodology of planetary systems. He made original contribution to astronomy by asserting the sun and the stars to be stationary, calculating the circumference of the earth, exposing the causes of eclipses and gravitation scientifically. In thirty verses of Gaṇitapāda, he has given the knowledge of arithmetic, algebra and geometry. This includes areas, volumes, trigonometry, shadow reckoning quadratic equation, rule of three sets, multiplication and division of fraction,

finding conjunction period of planets, rule of false position and so on.[44]

The seventh century is marked by the works of Brahmagupta who compiled Brāhmasphuṭa-sidhānta (Sindhind being its abridged version by Al-Khwārizmī) and Khaṇḍakhādyaka (translation in Arabic by Muhammad ibn Ibrāhīm al-Fazāri, c. +796, and Yáqūb ibn Ṭāriq, c. +796,). He applied arithmetic in shadow reckoning, series, interest, complex rule of three. He found rules for negative numbers, worked on indeterminate equations and trigonemetry of sun-dials. He discovered and solved the so called Pelcian equation, which is the basis of principles of quadratic equations. He also gave rules for finding areas and volumes of practical application, including cyclic quadrilateral.

Mahāvīrācārya (+9th century) went far ahead, generalizing his predecessor's work, giving rules for manipulation with zero, negative numbers identities, false position, linear equations and several types of problems. He was patronized by a Jaina king Amoghavarṣa. He found the area of an ellipse, and approached the solution of quadratic equations in a beautiful way. The remark of E. T. Bell is important, "The rule of signs became common in India after their restatement by Mahāvīra in the ninth century......The early history of complex numbers is much like that of negatives, a record of blind manipulations, unrelieved by any serious attempt at interpretation or understanding. The first clear recognition of imaginaries was Mahāvīra's extremely intelligent remark in the ninth century that, in the nature of things, a negetive number has no square root. He had mathematical insight enough to leave the matter there, and not to proceed to meaningless manipulation of unintelligible symbols."[45] His rules went beyond Diophantus and rediscovered by Leonardo Fibonacci regarding construction of right-angled triangles satisfying $x^2+y^2=a^2$.

Smith differentiates the work of Mahāvīrācārya from his predecessor and successor as follows, " ...For example, all of these writers treat of the

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44. For recent findings cf.
K. S. Shukla, Āryabhaṭīya, of Áryabhaṭa with the commentary of Bhāskara I and Someśvara, INSA, New Delhi, 1976.
K. V. Sarma, Áryabhaṭiya of Áryabhaṭa with the commentary of Sūryadeva Yajvan, INSA, New Delhi, 1976. K. S. Shukla and K. V. Sarma, Āryabhaṭīya of Āryabhaṭa, INSA, New Delhi, 1976.

45. Development of Mathematics, New York, 1945, pp. 173–175.
For a study of salient features of the work, cf. Jain, B. S., On the Gaṇitāsāra—Saṁgraha of Mahāvīra (c.850 A. D.), I. J. H. S., vol. 12, no. 1, May 1977, pp. 17-32.

areas of polygons, but Mahāvīrācārya is the only one to make any point of those that are reentrant. All of them touch upon the area of a segment of a circle, but all give different rules. The so called janya operation is akin to work found in Brahmagupta and yet none of the problems is the same. The shadow problems, primitive cases of trigonometry and gnomonics, suggest a similarity among these three great writers, and yet those of Mahāvīrācārya are much better than the one to be found in either Brahmgupta or Bhaskar, and no question is duplicated.[46]"

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46. op. cit., p. 75. For the work of his successor, Bhāskara II, cf. a recent article on "Some Mathematical Contributions of ancient Indian Mathematicians as given in the Works of Bhāskarācarya II, (12th cent. A. D.), I. J. H. S., vol. 12, no. 1, pp. 45-56, 1977.

    A series of papers by R. C. Gupta have been published in Mathematics Education, under the caption, Glimpses of Ancient Indian Mathematics, and perhaps still continuing. Cf. in the same series, Mahāvīrācārya on the Perimeter and Area of an Elipse, M. E., vol. VIII, no 1, 1974, Sec. B., 17-19. Cf. also his correction in, "Mahāvīrācārya's Rule for the Surface Area of a Spherical Segment : a new interpretation, Tulsi Prajña–2, 63–66.

CHAPTER 3.

A GLIMPSE OF MATHEMATICS IN THE TILOYA PANNTTĪ

The Tiloyapaṇṇatti belongs to the Karaṇānuyoga group of Prakrit texts to which belong the Jambūdīva saṃgaho, Loka vibhāga, Trilokasāra as also the upāṅga group texts, viz., Sūryaprajñapti,* Candraprajñapti, and Jambūdvīpa prajñapti. The contents of the texts are more or less the same as per their denomination.

The date of the text has been roughly estimated to be +5th century and its author is Yativṛṣabha who also compiled the cūrṇisūtras of the Kasāyapāhuḍa sutta which is a text on karma system theory belonging to c.+1st century. Except the description of the fourteen sequences (detailed in the Trilokasāra), this texts gives a systematic treatment of basic mathematics[47] applied to the karma mathematical theory. However, the text giving the introduction of traditional synopsis of the eternal cosmos, is not a mathematical text. It is full of propositional verses, relating results and formulas off and on. Scribes seem to have added material to it, perhaps upto +10th century. Alongwith the decimal place value notations there are certain symbols, and symbolic expressions for quantities ranging from numerate and innumerate to infinite, in syncopated form. Measure and mensuration have been correlated through long process of calculations from the ultimate unit to an observable unit. There are a few common features in the Dhavalā Commentary of Vīrasenācārya (c.+8th century) comparable with those in the Tiloyapaṇṇatti of Yativṛṣabha.

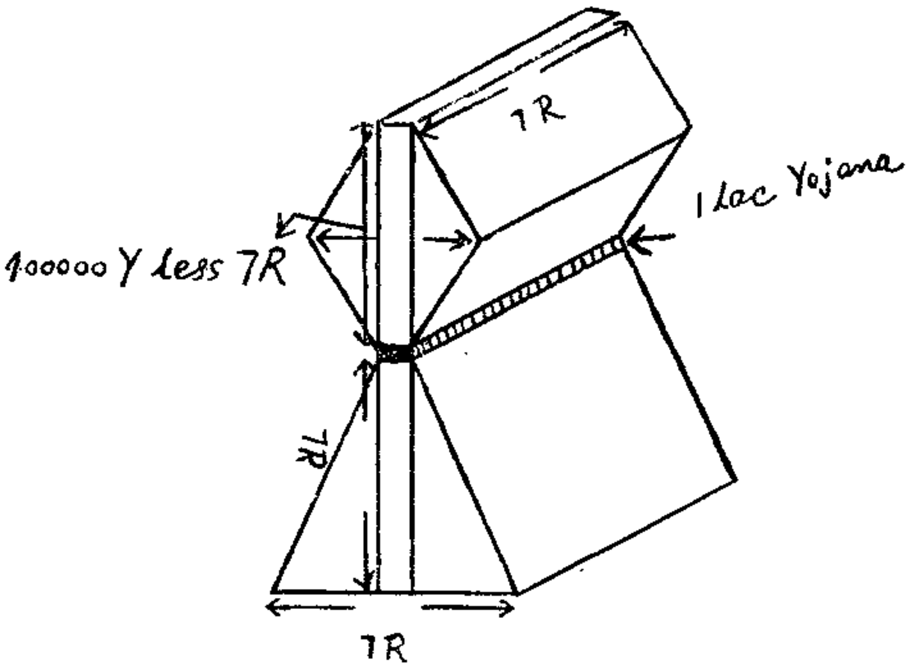
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*Thibaut, G., in, "Astronomie, Astrologie und Mathematik" published in "Grundriss dev Indo-Arishen Philologic und Altertumskunde" (Encyclopoedia of Indo-Aryan Research), vol. III, No. 9, p. 20, says that SP resembles strikingly with cosmological conceptions of Chau-pei, a Chinese Work.

47. Cf. Jain, L. C., Tiloyapaṇṇatti kā Gaṇita, Sholapur, 1958. Cf. also "The Mathematics in the first four Mahādhikāras of the Triloka prajñapti" by Sarasvati, T. A., The Jour. of G. J. R. I., Allahabad, vol. (xviii), Nov., 1961-Aug. 1962, pp. 27–51, for mathematical details.

(a) **Simile Measure :**

For the description of the three universes as shown in the labelled map in the margin, the units range from a map of the whole universe, comprising of the lower, middle and the upper, beyond which is the non-universe all around, known as empty space extending in all directions to the endlessly endless.



Space-point (pradeśa) to a rajju (rope). A point is defined to be the space occupied by an ultimate particle (parmāṇu). First a finger width is defined as follows[48] :

Endlessly endless paramāṇus	= 1 Uvasannāsanna skandha
8 Uvasannāsanna skandha	= 1 Truṭareṇu skandha
8 Truṭareṇu skandha	= 1 Trasareṇu skandha
8 Trasareṇu skandha	= 1 Rathareṇu skandha
8 Rathareṅu skandha	= 1 Bālāgra of uttama bhogabhūmi
8 Bālāgra of Uttama bhogabhumi	= 1 Bālāgra of of madhyama bhogabhūmi
8 Bālāgra of madhyama bhogabhūmi	= 1 Bālāgra of jaghanya bhogabhūmi
8 Bālāgra of jaghanya bhogabhūmi	= 1 Bālāgra of karma bhūmi
8 Bālāgra of karma bhūmi	= 1 Lika

48. Cf. T.P.G., pp. 18, 19.; T.P., vol. 1. 93–132.

8 Likas	= 1 Jūn
8 Jūns	= 1 Jau
8 Jaus	= 1 Aṅgula

This is a linear finger width whose symbol is 2 (two), also called utsedha sūchyaṅgula, used for measuring height etc. of body. Five hundred such utsedha aṅgulas make a pramāṅgula which is used to measure islands, oceans, rivers, mountains etc. Ātmāṅgula varies time to time and regions to regions and is used to measure small articles' number.

Further

6 Aṅgulas	= 1 Pāda
2 Pādas	= 1 Vitasti
2 Vitastis	= 1 Hātha
2 Hāthas	= 1 Rikku
2 Rikkus	= 1 Daṇḍa
1 Daṇḍa or 4 Hāthas	= 1 Dhanuṣa
	= 1 Mūsala
	= 1 Nālī
2000 Dhanuṣas	= 1 Krośa
4 Krośas	= 1 Yojana

Lishk and Sharma have evaluated the value of a Yojanạ to be about 6.37 miles.[49]

On the basis of cardinality two sets are connected as follow :

$$\log_2 \text{(Aṅgula)} = [\log_2 \text{(Palya)}]^2,$$

where the value of Addhā Palya in Samayas (instant) in given as follows :[50] {Jaghanyaparīta asaṃkhyāta}^{2} (sixtyone digits written in decimal place value notation) { $(2)^5$ $(15)^2$ $(38\frac{1}{2})^2$ $(7)^2$. madhyama Saṃkhyāta}.

The ultimate fraction of space is a pradeṣa, or a space-point. It is defined as the quantity of space occupied by a paramāṇu (ultimate particle), which is postulated to be indivisible and endowed with one taste, one colour, one odour as well as two nuances of touch.[51] An instant or Samaya corresponds to an instataneous occurence (vikāra) in a guṇa of a Dravya

49. Cf. Lishk, S. S. & Sharma, S. P., The Evolution of Measures in Jain Astronomy, Tīrthaṅkar, vol. 1, nos. 7–12, July–Dec. 1975, pp. 83–92.
50. Cf. T.P.G., p. 104, op. cit.
51. Cf. Akalaṅka, Tattvārthavārtikam, 5-8-3/4/5/6, pp. 449, 450, Pt. I, (1953), Pt II (1957), Kashi.

(fluent), and is the time taken by a paramānu which moves to the next point with minimum velocity.[52] The Samaya is also indivisible. Guṇa means control.

Further 1 Jagaśreṇī = 7 Rajjus (Rājū)
Loka = $(\text{Jagaśreṇi})^3$
= 343 cubic Rajjus

Loka is the universe, already defined.[53]

Again the number of space-points occupied in a Jagaśreṇī is given by

$$\text{Jagaśreṇi} = [\text{Ghanāṅgula}]^{\left(\frac{\log_2 \text{Palya}}{\text{Asaṁkhyāta}}\right)}$$

Where Ghanāṅgula is the cube of an Aṅgula. That is how the universe and all the fluents and their events are measured.[54]

So far as the practical time is concerned, its unit is Samaya. Now further time units are related with this indivisible unit as follows ;[55]

Asaṃkhyāta Samayas	=	1 Āvali
Saṃkhyāta Avalis	=	1 Ucchavāsa
7 Ucchavāsas	=	1 Stoka
7 Stokas	=	1 Lava
38½ Lavas	=	1 Nālī
2 Nālīs	=	1 Muhūrta
30 Muhūrtas	=	1 Dina
15 Dinas	=	1 Pakṣa
2 Pakṣas	=	1 Māsa
2 Māsas	=	1 Ṛtu
3 Ṛtus	=	1 Ayana
2 Ayanas	=	1 Varṣa
5 Varṣas	=	1 Yuga

Then proceeding this way through various units, the Acalātma unit of time is obtained which is equal to $(84)^{31} \times (10)^{90}$ years. This is then asked to be carried to Utkṛṣṭa Saṃkhayāta (Maximal Numerate). This will then be related with number measure.

52. Cf. T.P., Pt. I, vv. 284, 285, p. 176.
53. Cf. T.P.G., p. 18.
54. Cf. ibid., p. 22.
55. Cf. T.P.G., pp. 54–55.
For still bigger units of time related with change Dravya, Kāla, Kṣetra, Bhava, Bhāva, one may refer, "Reality"—Eng. Tran. of Pūjyapāda, "Sarvārtha Siddhi", by S. A. Jain, Calcutta, 1960.

The earlier above is the description of simile measure in Jainology, with the help of which space and time measures are given in terms of cardinal numbers of sets.

(b) Number Measure :

For a modern mathematician, there is no need of a well-defined word "number". However it could be stated generally that a number is a set of equivalent sets alone. This would include all types of rational, transcendental, irrational, natural and integer numbers.

During the Pythagorean era, discussions on infinity had been initiated, but arguments or paradoxes of Zeno (+5th century) had created fear in moving towards analysis of infinities. After Galileo (+1564 to +1642) when George Cantor (+1845 to +1918) started his work on infinities and sets, the mathematician began to say that this subject has been brought 100 years too early.[56] Whereas, in India this had reached its climax even before the early centuries of Christian Era.[57]

The Jaina School needed a quantitative analysis of Karmic events, for which new mathematical but proper infinity was to play an important role. Through a long process, twentyone types of ranges were produced by carrying on certain types of multiplication, squaring as well as by adding or projecting various types of sets (in cardinal or in ordinal lay out). These ranges gave twenty one types of divisions of new mathematical infinity ranging from unity to the cardinal number of the indivisible-corresponding-sections of Omniscience (all knowledge or Kevalajñāna).[58]

1. Saṃkhyāta (Numerable)
   - 1.1. Jaghanya (Minimal)
   - 1.2. Madhyama (Intermediate)
   - 1.3. Utkṛṣṭa (Maximal)

2. Asaṃkhyāta (Innumerable)
   - 2.1. Parīta (Extended)
     - 2.1.1. Jaghanya
     - 2.1.2. Madhyama
     - 2.1.3. Utkṛṣṭa
   - 2.2. Yukta (Yoked)
     - 2.2.1. Jaghanya

56. Cf. Fraenkel, A. A, Abstract Set Theory, Amsterdam, 1953.
57. Jain, L. C., Set Theory in Jaina School of Mathematics, I. J. H. S., Vol. 8, nos. 1 & 2, 1973, 1–27.
58. Cf. T.P.G., pp. 55 et. seq.

2.2.2. Madhyama
2.2.3. Utkṛṣṭa

2.3. Asaṃkhyāta
2.3.1. Jaghanya
2.3.2. Madhyama
2.3.3. Utkṛṣṭa

3. Ananta (Infinite)

3.1. Parīta
3.1.1. Jaghanya
3.1.2. Madhyama
3.1.3. Utkṣṛṭa

3.2. Yukta
3.2.1. Jaghanya
3.2.2. Madhyama
3 2.3. Utkṛṣṭa

3.3. Ananta
3.3.1. Jaghanya
3.3.2. Madhyama
3.3.3. Utkṛṣṭa

Two important features of the above classification are (1) the existence of innumerability between the domains of numerability and infinity (2) the existence of infinity greater than another infinity.

Quoting Galileo's work on "continnum of divisibles", as well as on the "Theory of Real Numbers" of Cantor, Bell[59] remarks.

"Salv.—I see no other decision that it may admit, but to say, that all Numbers are infinite; Squares are infinite; and that neither is the multitude of squares less than all Numbers, nor this greater than that : and in conclusion, that the Attributes of Equality, Majority, and Minority have no place in Infinities, but only in terminate quantities......" Further, 'Resolving Simplicius' doubt about the conceit of 'assigning an Infinite bigger than an Infinite' Cantor proceeded to describe any desired number of such bigger Infinities...... For cardinal numbers also Cantor described 'an Infinite bigger than an infinite' to confound the Simpliciuses...... He proved (1874) that the class of all algebraic numbers is denumerable, and gave (1878) a rule for constructing an infinite non-denumerable class of real numbers.

59. Cf. Development of Mathematics, p. 273.

Were we to make a list of spectacularly unexpected discoveries in mathematics, there two might head our list."[60]

In the Tiloyapaṇṇatti, innumerate is the domain of clairvoyance, whereas infinite is the domain of Omniscience. The domain of Scripture-Omniscience is only the maximal numerate.[61] For actual innumerability, innumerate type of set is to be mixed, and for attainment of actual infinity, infinite type of set is to be mixed to the last number obtained by defined processes.

(c) Approach by Exhaustion :

In the cosmography of Jaina School, the circumference of the Jambū island is taken to be a circle of diameter 100,000 yojanas. The texts like the Anuyogadvāra Sūtra, the Trilokasāra etc., mention the value of this circumference to be 316227 yojanas, 3 Krośas, 128 daṇḍas and 13½ aṅgulas nearly.

Whereas, it was not known how the author of the Tiloyapaṇṇatti gave a value from the Diṭṭhivāda upto a very fine unit of length called avasannāsanna skandha, where 8^{12} of these units made one aṅgula. Gupta[62] has recently showed that the approximation to $\frac{23213}{105409}$ kha kha was computed by making use of the two following rules :

(i) $\text{Circumference} = \sqrt{10\ (\text{diameter})^2}$

(ii) $\sqrt{a^2+x} \quad = a + (x/2a)$ ..

According to Gupta, "Naturally, we are keen to know the cause of of disagreement between the two sets of values, particularly because the values are intended to give accuracy to a very fine degree of smallness. Is there some arithmetical error of calculation in extracting the square root, successively, to the desired degree ? Or, the Jainas followed some different procedure ?"

The approximation was known to the Greek Heron of Alexandria (between +c.50 to +c.250 A. D.) and even to the ancient Babylonians. Chinese Sun Tzu (+c.280 to +c.473), also quoted a result bearing the same type of approximation.

The above results show the heights and depths of the mathematical manoeuvre in the school.

60. Cf. ibid., p. 275.

61. Cf. T.P.G., pp. 1, 2, 54–62.

62. Gupta, R. C., Circumference of the Jambūdvīpa in Jaina Cosmography, I. J. H. S., May 1975, vol. 10, no. 1, pp. 38–46.

CHAPTER 4.

CERTAIN PECULIARITIES IN THE MATHEMATICS OF DHAVALĀ

(a) Place Value :

Singh[63] notes that the author of the Dhavalā, Vīrasenācārya, was fully conversant with the place value system of notation. E.g. :

(i) 79999998 is expressed as a number which has 7 in the beginning, 8 at the end, and 9 repeated six times in between.

(ii) 46666664 is expressed as sixty-four, six hundreds, sixty-six thousands, sixty-six hundred-thousands, and four koṭis.

(iii) 22799498 is expressed as two koṭis, twenty-seven, ninety-nine thousands, four hundred and ninety-eight.

In Jaina literature the method (i) is found elsewhere also. It shows familiarity with the place value notation. In (ii) the smaller denominations are expressed first. This is not in accordance with the general practice current in Sanskrit literature. Similarly, the scale of notation is hundred and not ten as is generally found in Sanskrit literature. In Pali and Prakrit, however, the scale of hundred is generally used. In (iii) the highest denomination is expressed first. Quotations (ii) and (iii) are evidently from different sources.[64]

(b) Indices :

At a place Dhavalā[65] states a number to lie between the sixth-square of two and the seventh-square of two; or to be more precise, it is said to lie between Koṭi-Koṭi-Koṭi and Koṭi-Koṭi-Koṭi-Koṭi, that is between

$$2^{2^{6}} \quad \text{and} \quad 2^{2^{7}}$$

63. "Mathematics of Dhavalā", op. cit.

64. Cf. ibid.

65. Cf. Dhavalā, III, p. 253, op. cit.

Actually, this number, occupying twenty-nine notational places is 79, 22, 81, 62, 51, 42, 64, 33, 75, 93, 54, 39, 50, 336, based on the calculated area of inhabitants of the world human beings, showing inconsistency of other views, as shown by the author.

The theory of indices as described in the Dhavalā is to some extent different from other mathematical works. This theory appears to be primitive and in use before + 500. The basic ideas seem to be those of (i) the square, (ii) the cube, (iii) the successive square, (iv) the successive cube, (v) the raising of a number to its own power, (vi) the square root, (vii) the cube root, (viii) the successive square root, (ix) the successive cube root, etc.

For example, the third vargita saṃvargita of 2 may be found to be $(256)^{256}$. The operations of such duplation and mediation were recognized by the Egyptians and the Greeks but there is no trace of them in the Indian mathematical works :

$$2^{2^7} \div 2^{2^6} = 2^{2^6}$$

(c) Logarithms :

Singh[66] recognized the Ardhaccheda of a number as logarithm to the base two. Thus (Ardhaccheda of 2^m) = m. Similarly he recognized the function of a function in finding Vargaśalākā of Dhavalā as logarithm to the base two of the logarithm of a number to the base two. Thus Vargaśalākā of 2^{2^m} could be noted to be m. Similarly Trakaccheda of a number was found to be equal to logarithm to the base three, and so on.[67]

The following table gives the results known to the author of the Dhavalā.[68]

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66. Cf. Mathematics of Dhavalā, op. cit. In this connection. Kapadia, H. R., appears to have recognized already that if $x=2^n$, then n is called the ardhaccheda of x. He reminds that if $a^x = n$, x is called the logarithm of n to base a : Cf. Gaṇita Tilaka, Baroda, 1937, p. xxv (intro.).

67. T. Heath writes, "The Neo-Pythagoreans improved the classification thus. With them the 'even-times even' number is that which has its halves even, and so on till unity is reached," A History of Greek Mathematics, Oxford, (Pts. I & II, 1921), Pt. I, p. 72.

68. Cf. Jain, L. C., On certain Mathematical Topics of the Dhavalā Texts, I. J. H. S., Vol. 11. no. 2, 1976, 85-111.

(1) $\log ({}^{a}/_{b}) = \log a - \log b$

(2) $\log (a.b) = \log a + \log b$

(3) $\log 2^{c} = c,$ where base is 2,

(4) $\log (a^{a})^{2} = 2 a \log a$

(5) $\log \log (a^{a})^{2} = \log a + 1 + \log \log a$

(6) $\log (a^{a})^{a^{a}} = a^{a} \log a^{a}$.

Further, if we donote the

1st vargita-saṃvargita of $x = x^{x} = \overline{x}|^{1}$

2nd vargita-saṃvargita of $x = \overline{\overline{x}|^{1}}|^{1} = \overline{x}|^{2}$

3rd vargita-saṃvargita of $x = \overline{\overline{x}|^{2}}|^{1} = \overline{x}|^{3}$,

Vīrasena shows

$\log_2 \log_2 \overline{x}|^{3} < \{ \overline{x}|^{1} \}^{2}$

$\log_2 \overline{x}|^{3} = \overline{x}|^{2} \log_2 \overline{x}|^{2}$ and so on.

It may be noted that the theory of logarithms was discovered by Baron Napier (+ 1550 to + 1617) and J. Burgi (+ 1552 to 1632). Sanford remarks, "The discovery of logarithms, on the other hand, has long been thought to have been independent of contemporary work, and it has been characterized as standing isolated, breaking in upon human thought abruptly without borrowing from the work of other intellects or following known lines of mathematical thought."[69]

(d) Fraction-manipulation :

The examples used by Vīrasena show the relics of an age when division was considered in terms of set-theoretic approach, because the dealings were in terms of rāśis : [70]

(1) $$\frac{a^{2}}{a \pm ({}^{a}/_{b})} = a \mp \frac{a}{b \pm 1}$$

(2) If q and r are the quotients when a is divided by b and c, then

69. A Short History of Mathematics, p. 193.

70. Cf. Mathematics of Dhavalā, op. cit.

$$\frac{a}{b \pm c} = \frac{r}{(r/q) \pm 1} = \frac{q}{1 \pm (q/r)}$$

(3) If $\frac{a}{b} = c$, then

$$\frac{a}{b + \frac{b}{n}} = c - \frac{c}{n+1}$$

$$\frac{a}{b + d} = c - \frac{c}{\frac{b}{d} + 1}$$

and so on.

(e) Analytical Methods :

Eight analytical methods[71] were basically applied by Vīrasena to sets' knowledge apart from various other methods. They were pramāṇa (measure), kāraṇa (reason), nirukti (explanation), vikalpa (abstraction or extra-creation), khaṇḍita (cut), bhājita (division), viralana (spread) and apahṛta (removal).

The method of vikalpa[72] shows that the author was conversant with the laws of indices and exponentiation for finite and transfinite sets. For example, the set of souls in hell, having mythic outlook is illustrated through a lower vikalpa of a dyadic sequence :

$$L(F)^{3/4} = L \,.\, (L)^{1/2} . (L)^{1/4} .... (L)^{\frac{1}{A}} \left[(L)^{\frac{1}{A}} \div \frac{L}{(F)^{3/4}} \right]$$

Similarly,

$$T^2 \div n^{\left\{ \log_n \left(\frac{T^2}{g} \right) \right\}} = g.$$

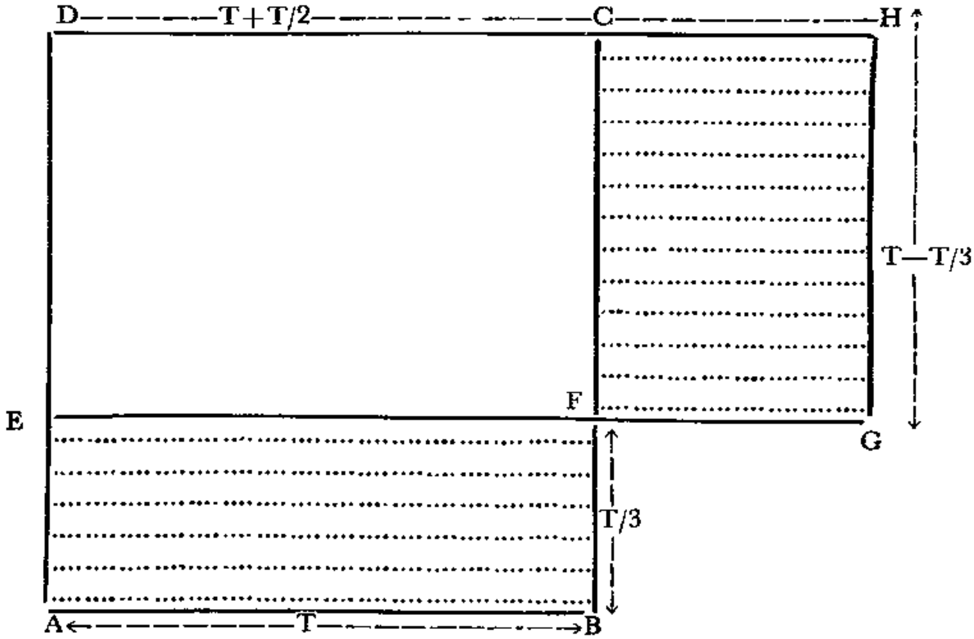
The method of Kāraṇa[73], has been given as a geometrical representation of an algebraic equation, similar in style to the Greek's method of application of areas :

71. Cf. Jain, L. C., On the Jaina School of Mathematics, C. L. Smriti Granth, Calcutta, 1967, pp. 265-292.

72. Cf. ibid.

73. Cf. ibid.

$$T^2 \div \left[T + \frac{T}{2}\right] = \frac{2}{3}T = \left[T - \frac{T}{3}\right]$$



Here the figure ABCD represents T^2, the rest is obvious from the figure. The method is thought by Heath to have been originated with the Pythagoreans. From Babylonian records, however, it appears that there might have been a diffusion of such knowledge from the Near East to Greece in a period close to the eve of the Macedonian offensive against the Persian Empire.[74]

(f) A new value of π

Apart from the two values of π as 3 and $\sqrt{10}$, Vīrsena has quoted from some earlier work the value of π in the following form :

If

Circumference = C

Diameter = D ,

then $C = 3D + \frac{16D + 16}{113}$ was the formula according to the

74. Cf. Neugebauer, op. cit., p. 149, et seq.

couplet.

Vīrsena, interprets the above as

$$C = 3\,D + \frac{16\,D}{113} = \frac{355\,D}{113}$$

which gives $\pi = \frac{355}{113}$.

Singh[75] remarks, "the term 'Sahitaṁ' was used in the sense of addition as well as 'multiplication', i.e., repeated addition, a number of times in the Vedāṅga Jyotiṣa but has been used in that double sense by Āryabhaṭa (c.+499) and succeeding mathematicians. This leads one to conclude that the above quotation is from some work written before the fifth century A. D., when "Sahitaṁ" was being used in both the senses of multiplication as well as addition. It would appear, therefore, that the so called Chinese value of $\pi = \frac{355}{113}$ was also known in India and was perhaps used in India earlier than in China. It might be that the Chinese got this value from India, through Buddhist missionaries or perhaps they found out the value independently." According to him, "Another noteworthy feature in the above quotation is the remark 'finer than the fine'; from this it follows that a 'fine' value of π was already known. This fine value of π may have been $\sqrt{10}$ or $\frac{22}{7}$. In the latter, the connection with Āryabhaṭa's value is obvious—the third convergent being a close approximation than the second."[76]

(g) An infinite process

For disproving a prevalent theory regarding the shape of the universe, Vīrsena found the volume of frustum of a cone by an infinite process of cutting it. Out of the frustum he first cut out a cylinder from it and stretched the remaining figure into a long prism which could again be cut into tetrahedrons of various volumes tending to zero area. From the point of view of history of mathematics, Singh[77] could find out the following items of interest :

(i) It was assumed that a body with curved boundaries could be deformed into another with plane boundaries in such a way that its volume remained unchanged. In particular, if the hollowed out cone of figure is deformed which has plane boundaries, then the volume remains unchanged.

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75. History of Mathematics in India from Jaina Sources, The Jaina Antiquary, Vol. xvii, no. ii, Dec. 1950, Arrah, pp. 54–69.

76. Cf. ibid.

77. Cf. ibid.

(ii) The principle of construction for the purpose of demonstration or proof had been assumed to hold true, for finding the volume of the tetrahedrons.

(iii) The formula

$$\text{Sum} = \frac{\text{first term}}{1 - \text{Common ratio}}$$

when the common ratio is less than unity
for the geometric progression

$$\begin{aligned}\text{Sum} = \text{first term} &+ (\text{first term})\,(\text{common ratio}) \\ &+ (\text{first term})\,(\text{common ratio})^2 \\ &+ \ldots\ldots\ldots\ldots\end{aligned}$$

had been assumed.

(iv) For the purpose of finding the value of the volume of the above solids, π was assumed to be $\frac{355}{113}$.

(v) If D is the diameter of the lower base and d the diameter of the upper base of the frustum, and h its height, the volume can be obtained by finding the sum of the following series :[78]

$$\begin{aligned}\pi\left(\frac{d}{2}\right)^2 h &+ \left(\pi \,.\, d \,.\, h \,.\, \frac{D-d}{2^2}\right) \\ &+ \left(\pi\,\frac{D-d}{2^2} \,.\, \frac{h}{2} \,.\, \frac{D-d}{2}\right) \\ &+ \left(\pi\,\frac{D-d}{2^3} \,.\, \frac{h}{2^2} \,.\, \frac{D-d}{2}\right) + \ldots\ldots\end{aligned}$$

This type of infinite process has been denoted an innumerate type of process by Vīrsena.[79] It comprises of sequence of construction which is innumerate and the sum of an infinite series is practically applied. It is not known what was the justification for making use of such infinite processes, which due to end in exhaustion could be denominated as innumerate.

78. Cf. Mathematics of Tiloypaṇṇatti, p. 15, op. cit.

79. Cf. Ṣaṭkhaṇḍāgama, book 4, p. 15.

CHAPTER 5

CERTAIN COMMON TOPICS OF BASIC MATHEMATICS

The Śvetambara texts form the major portion of the knowledge of entire Jaina canon and works available are the first eleven añags, upāṅgas, prakīrṇakas, cheda sūtras, mūla sūtras, cūlikā sūtrās. The Digambara texts recognize only a very minor part of the twelfth aṅga, Dṛṣṭivāda, constituting works like the Ṣaṭkhaṇḍāgama and the Kasāyapsāhuḍa sutta, as well as their commentaries. Thus the two traditions mutually supplement each other's knowledge, perhaps to a certain extent[80].

The Śvetambara collection of the Jaina canonical literature comprising of eleven aṅgas and fourteen pūrvas in fragmentary form was effected in the 3rd century B. C. by a council of monks at Patliputra. The second council was organized for the same purpose in – 150 at Kumārīparvata near Bhuvaneshwar. The third council was organized in + 66, the fourth in + 300 and the fifth in + 466 at various places.

The Digambara tradition believes in the verbal transmission of knowledge of original twelfth aṅga from preceptor to disciple till the present recension of the minor works was redacted.

(a) Topics :

Sthānāṅga-sūtra, mentions ten topics in mathematics of Saṁkhyāna or calculation :

1. Parikarma (fundamental operations),
2. Vyavahāra (subjects of treatment),
3. Rajju ("rope", perhaps geometry),
4. Rāśi ("set" or "heap", trai-rāśika),
5. Kalāsavaṇṇa (fraction),
6. Yavat-tāvat ("as many as" or 'simple equations' or similar processes),

80. Cf. Jain, J. C., Prākrita Sāhitya Kā Itihāsa, 1961 E

7. Varga (square or quadratic equations),
8. Ghana (cube or cubic equations),
9. Varga-varga (square of square or biquadratic equations),
10. Vikalpa (abstraction or permutation and combination).

From the earlier treatment it will be manifest that the above ten topics alongwith alpabahutva (comparability), which has, unfortunately, not been included above, the mathematical philosophy of Karma could well go on. Sequences and comparability could be included in topic of "set" or "rāśi" which is a word used very frequently, specially in the Dhavalā commentary, for every quantity or quality there was a number associated with it.

Datta ventured to presume that the term yāvat-tāvat was connected with the Rule of False Position, which, in the early stage of algebraic science in every country, was the only method of solving linear equations.[81] He further remarks:"[82] "That term enters largely into Hindu Algebra of later times as the symbol for the unknown. It has been suggested that it is connected with the definition of the unknown quantity by the Greek Diophantus (c.75 A.D.) as "containing an indeterminate or undefined multitude of units (pléthos monàdon aoriston)". The implication behind that suggestion was to show the Greek influence in the Hindu Algebra. It is now found that yāvat-tāvat has entered into Hindu mathematics more than five centuries before Diophantus. So if that suggestion be at all true, though I doubt it, it will have to be admitted that the balance of evidence is in favour of the Hindus, showing the possibility of the Greek Algebra being influenced by the Hindu Science. This will take aback the author of that suggestion."

(b) Place value notation for subtraction :

The Jaina School is found to use a place value notation for subtraction from factors of numerical or algebraic quantities. This notation[83] has been used in the Tiloypaṇṇaṭṭi. It has been used profusely in the commentaries of the Gommaṭasāra. Ṭoḍarmala,[84] one of the commentators, has exposed the use in the following manner :

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81. The Jaina School of Mathematics, op. cit.
82. Cf. ibid. Cf. also G. R. Kaye, Indian Mathematics, Calcutta, 1915, p. 25. Cf. also, Jaina L. C., On the Jaina School of Mathematics, op. cit.
83. Cf. TP, pt. II, p. 609, op. cit.
84. Cf. T Arthasaṃdṛṣti of Gommaṭasāra, Gandhi Haribhai Deokaraṇa Granthmala, Calcutta, c. 1919.

Let L | 5 | 4 | 3 denote the product of L, 5, 4 and 3. If L or one lac is subtracted from this product the remaining quantity is written as

$$L\ \overset{1}{5}\ \overset{\text{c}}{\overline{|\ 4\ |\ 3}}$$

The quantity less 5 lacs is denoted as

$$L\ |\ 5\ |\ \overset{1}{4}\ \overset{\text{c}}{\overline{|\ 3}}$$

The quantity less 20 lacs is denoted as

$$L\ 5\ |\ 4\ |\ \overset{1^{\text{c}}}{3}$$

The quantity less 3 lacs is denoted as

$$L\ \overset{3^{\text{c}}}{5}\ |\ 4\ |\ 3$$

The quantity less 12 lacs is denoted as

$$L\ \overset{1^{\text{c}}}{5}\ |\ 4\ |\ 3$$

The quantity less 15 lacs is denoted as

$$L\ 5\ |\ \overset{1^{\text{c}}}{4}\ |\ 3$$

The quantity less 30 lacs is denoted as

$$L\ 5\ |\ \overset{2^{\text{c}}}{4}\ |\ 3$$

This type of place value has been used for fractional as well as other types of complicated quantities. Thus the credit goes to this school for being able to follow the way of place value notation for zero, which was also used for subtraction notation[85]. For example, the expression for one lac less one could be denoted as

$$\begin{array}{c}0\\1\\L\end{array} \quad \text{or} \quad \begin{array}{cc}L & 0\\ & 1\end{array}.$$

In the above ᶜ stands for minus. Long before, in the Tiloypaṇṇatti, रिण word was in use for minus and it is just possible that it was cut short as रि and further as ि and then as ᶜ.

85. Cf. ibid, p. 9.

Regarding the development of the place value system of notation, Datta[86] finds, "We can not say what were the forms of the numerals used by the early Jainas. That they had some numeral characters, we have no doubt. For as early as the fourth or fifth century before the Christian era, we find in a list enumerating the different written characters (lipi) known about that time, the mention of aṅkalipi and gaṇita-lipi.[2] That list has been reproduced in the Prajñapanā-sutra[3] of Syâmârya who died in 376 A. V. (= 92 or 151 B.C.). There two names suggest further that the forms of numerals used for different purposes were different. The former refers to the numerals used in engraving and the latter to those used in ordinary writing. In the Jaina literature, as also in the Vedic literature we ordinarily find that a distinction is made between form of alphabets used in engravings, (called by the Jainas Kāṣṭakarma or "wood work") and in manuscripts, (called pustaka-karma or "book work").[4] This reference is very important in as much as it shows how one-sided and partial are the views of those writers who consider the origin and development of the Hindu numerals on the palaeographic evidence only."

In the Viśeṣāvaśyakabhāṣya (v. 704) of Jinabhadra Gaṇi, a verse of Bhadrabāhuṡ Āvaśyakaniryukti is quoted as

> "Thibugāgāra jahanno vaṭṭo
> ukkosamāyao kiñci".

Hemcandra Sūri opines that thibuga signifies "bindu". Datta asked Kapadia[87] whether "Is it then the 'zero' of the decimal place-value notation? If so, it will have to be admitted that the modern place-value notation was known in India in the 4th century before the Christian era". Kapadia[88] further advances the following points in this regard,

(1) In the 142nd sūtra of Anuyogadvāra there is a reference to sthāna (thāṇehiṁ) or places of decimal notation.

(2) A denominational name like koṭa koṭi (koḍa koḍi) is mentioned in this sūtra as indicating its connection with places of numeration.

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86. Cf. The Jaina School of Mathematics, op. cit.

87. Cf. Gaṇita Tilaka by Śripati, op. cit., p. xxi, intro.

88. Cf. ibid., p. xxii.
In the Tiloyapaṇṇaṭṭi, acalātma denoting $(84)^{31}$ $(10)^{90}$ years has been expressed as 84 | 31 | 90. Here 31 stands for the product of 84 to be effected 31 times into itself, and 90 stands for 90 zeros to be placed after the term as place values. (Cf. T.P., I. v. 4. 308, p. 178).

(3) A very big number extending to 29 places, i.e. to say a number consisting of 29 digits is here referred to as we shall shortly notice.

(4) Vyavahāra sūtra (uddeśaka I) furnishes us with a term gaṇana-sthāna for place of calculation, so says Dr. Datta.

(c) **Permutations and Combinations :**

The early Jaina literature shows that the school had a great liking for this subject studied as bhaṅga or vikalpa (abstraction), in form of gaṇita. This was also applied by early Hindu writers in the field of philosophy, medicine, astrology and so on. Instances are found in the Bhagbatī sūtra (c.+300), when speculation is made about different philosophical categories arising out of the combination of n fundamental objects one at a time (eka-saṁyoga), two at a time (dvika-saṁyoga), three at a time (trika-syṁyoga), or more at a time.[89]

Vīrsena[90] quotes a verse where the total number of syllables contained in śrutajñāna is 1, 84, 46, 74, 40, 73, 70, 95, 51, 615.

This is based on the fact[91] that there are 34 vowels and 30 consonants from which total number of syllables which could be formed is $2^{64}-1$. Bhagabatī sūtras 341 and 371 – 374 further form out of different senses (Karaṇas), or selections made out of males and females and eunuchs or combinations and permutations of various other things which could be put in the prevalent form as

$$^{n}C_{1} = n, \quad ^{n}C_{2} = \frac{n(n-1)}{\underline{|2}} \quad ,\ldots\ldots\ldots$$

$$^{n}P_{1} = n, \quad ^{n}P_{2} = n(n-1) \quad ,\ldots\ldots\ldots$$

Śīlāṅka (c.+862), the commentator quotes three rules regarding permutations and combinations.[92] Two of them are in Sanskrit and Ardha Māgadhī. The first is for determining the total number of transpositions that can be made out of a number of objects (bhedasaṁkhyā-parijñanāya).[93]

89. Cf. Bhagabatī–sūtrā (v. 314).

90. Cf. Jain, On the Jaina School of Mathematics, op. cit.

91. Cf. Bhagabatī-Sūtrā ,

92. Vide the commentary on Sūtrakṛtāṅga-sūtra, samayādhyayana, and anuyogadvāra. Cf. also, "Mathematics of Nemicandra" by B. B. Datta, The Jaina Antiquary, I, no. ii, (1935), 25-44.

93. "Beginning with unity upto the number of terms, multiply continually the (natural) numbers : This is to be known as the vikalpa-gaṇita."

This represents 1 . 2 . 3 . ...(n – 1) n . Spreading was known as prastāranaya naopāya. The translation of the Sanskrit verse runs as :

"The total number of permutations being divided by the last term, the quotient should be divided by the next : They should be successively placed by the side of the initial term in the calculation of permutations and combinations."

The above may be explained as follows :

The total number of permutations of n things taken all at a time =1 . 2 . 3......(n).

The total number of permutations having the nth thing for beginner,

$$= \frac{|\underline{n}}{n} = |\underline{n-1} \ .$$

Similarly other types could be found.

Nemicandra[94] gives full details of the method of finding combination or combinatorial specific term. He explains five terms to be understood in the mastery of the process,

(i) Saṃkhyā (number types of ālāpas)

(ii) Prastāra (extension or placing of number in order)

(iii) Parivartana (the order in which one could get on from one type to another)

(iv) Naṣṭa (finding out the type with the help of its ordinal number)

(v) Samuddiṣṭa (finding out the ordinal number by placing the type).

The same method is applied for finding out the combinations in relation to pramāda and śīla. Eighteen thousand types of śīla (character) are elaborated in the above style as follows :[95]

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94. Gommaṭasāra, op. cit., Jīvakāṇḍa, vv. 34–44 and 65. Cf. also Bag, A. K., Bionomial Theorem in Ancient India, I. J. H. S., vol. 1, no. 1, 1966, pp. 68–74.

95. The total kinds become $10 \times 10 \times 5 \times 3 \times 3 \times 2 \times 2 = 18000$. This is the method of prasāra.

Details about this method are available in Śrī Prastāra Ratnāvalī by Muni Ratnacandra, Bikaner, (Vik. Sam. 1981). It deals with permutations and combinations.

YOGA	Y_1 1	Y_2 2	Y_3 3							
KARAṆA	K_1 0	K_2 3	K_3 6							
SAṂJÑĀ	S_1 0	S_2 9	S_3 18	S_4 27						
INDRIYA	I_1 0	I_2 36	I_3 72	I_4 108	I_5 144					
JĪVA	J_1 0	J_2 180	J_3 360	J_4 540	J_5 720	J_6 900	J_7 1080	J_8 1260	J_9 1440	J_{10} 1620
DHARMA	D_1 0	D_2 1800	D_3 3600	D_4 5400	D_5 7200	D_6 9000	D_7 10800	D_8 12600	D_9 14400	D_{10} 16200

(d) Sequences and Progressions

It is only Trilokasāra which gives information about the dyadic type of sequences which pass through various types of finite and transfinite cardinals of sets.[96]

The fourteen types of sequences are as follows :

1.	Sarva	1, 2, 3, 4, 5, ...
2.	Sama	2, 4, 6, 8, 10, ...
3.	Viṣama	1, 3, 5, 7, 9, ...
4.	Kṛti	$1^2, 2^2, 3^2, 4^2, 5^2$, ...
5.	Akṛti	2, 3. 5, 6, 7, ...
6.	Ghana	$1^3, 2^3, 3^3, 4^3, 5^3$, ...
7.	Aghana	2, 3, 5, 6, 7, 9, 10, ...
8.	Kṛti Mātṛka	1, 2, 3, ...upto $\sqrt{x}$.
9.	Akṛti Mātṛka	$\sqrt{x}+1, \sqrt{x}+2$,......, x.
10.	Ghana Mātṛka	$(1^3)^{\frac{1}{3}}, (2^3)^{\frac{1}{3}}$,...... upto the last cubable number before y.
11.	Aghana Mātṛkā	$y+1, y+2, y+3, y+4$, ...; x.
12.	Dvirūpa Varga	$2^{2^1}, 2^{2^2}, 2^{2^3}$, ...
13.	Dvirūpa Ghana	$2^{3.2^0}, 2^{3.2^1}, 2^{3.2^2}$, ...
14.	Dvirūpa Ghanāghana	$2^{3^2.2^0}, 2^{3^2.2^1}, 2^{3^2.2^2}$, ...

The Jaina School, in every one of its cosmographical sets, describes innumerable concentric rings of alternate continents and oceans, the diameter of each of the successive rings is double that of its predecessor ring. The diameter of the nth ring could be found to be 2^{n-1}, by assuming that of the first to be unity.[97]

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96. Cf. Jaina, L. C., Divergent sequences locating transfinite sets in Trilokasāra, I. J. H. S., Vol. 12, no. 1, May 1977, 55–75.

97. Cf. T. P. G., pp. 69–75. Areas could also be compared in various details of the islands and oceans.

Progressions have also been detailed in the Tiloypaṇṇatti[98] as well as Gommaṭasāra, Trilokasāra and earlier texts. They are either arithmetical progressions or geometrical series. The general terminology of the arithmetic progression is as follows :

First term : Ādi, Mukha, Vadana, Prabhava,

Common Difference : Caya, Uttara,

Number of terms : Gaccha, Pada,

Sum : Saṅkalita, Dhana.

The general formula[99] for the sum, transformed in various forms is

$$\text{Sum} = \frac{\text{Gaccha}}{2}\left\{2\ \text{first term} + (\text{Gaccha}-1)\ \text{common difference}\right\}$$

For the geometrical progression,[100] common ratio is called guṇakāra, and its general formula has been given as

$$\text{Sum} = \frac{\left\{(\text{common ratio})^{\text{Gaccha}} - 1\right\}\text{first term}}{\text{Common ratio} - 1}\ .$$

Saraswathi remarks, "Arithmetical progressions find full treatment with separate formulae for finding the first term (a), the common difference (d), the period (n) and the sum (s). The series which occasions these rules, is a rather complex one of 49 terms with 7 groups within it, which themselves are separate progressions whose periods again form another arithmetical series. The expression

$$S = n\left[a + \left(\frac{n-1}{2}\right)^2 d + \frac{n-1}{2} - d\right]$$

for the sum of any group within the series is a distinct contributions of the TP to series Mathematics. The treatment of geometrical progressions is comparatively meagre."[100-A]

The same type of series appear in the description of three operational activities in Jaina philosophy working at various control stations affecting Karmic output as well as state existence.[101] The guṇahānis (geometric regressions) are such that their elements in particles decrease in

98. Cf. T. P. G., pp. 41-48.

99. Cf. ibid., p. 43.

100. Cf. ibid., p. 48.

100-A. Cf. The Mathematics in the first......Trilokaprajñapti, op. cit., p. 51.

101. Cf. Gommaṭasāra, Jīvakāṇḍa.

common ratio, whereas their energy-levels increase by unity. Similar phenomena happen to be in Yoga station structures in forms of varga, vargaṇā, spardhaka, guṇahāni etc.[102] If a varga is denoted by v, energy-level by s, d as the common difference, vargaṇā by W, then the first spardhaka of the first geometric regression or guṇahāni is given by

$$W_{v}^{s} + W_{v-d}^{s+1} + W_{v-2d}^{s+2} + \ldots\ldots\ldots + W_{v-(s-1)d}^{2s-1}$$

with the help of the superscripts and subscripts.

The first spardhaka of the second guṇahāni may be written in the following form

$$W_{\frac{v}{2}}^{(n+1)s} + W_{\frac{v}{2}-\frac{d}{2}}^{(n+1)s+1} + \ldots\ldots\ldots + W_{\frac{v}{2}-(s-1)\frac{d}{2}}^{(n+1)s+s-1}$$

Thus it appears that the study of complicated forms of sequences and progressions were the first requisits of the Jaina Karmic philosophy and theory of cosmological structures, round the advent of Christian era.[102A]

(e) Mensuration

The geometry of a circle and a straight line is a geometry of the Jambū island and its symmetric mountains. Moreover the conics are the outcome of various sections of the cone subtended at the eye of an observer by the paths of various heavenly bodies in all of their aspects.

Thus the relations between various parts for the unknowns became a subject of intensive study since the philosophy in India took a turn, some centuries before Christ. The new philosophy was required to explain the effect of action in all its phases, and alongwith the past knowledge, a scientific quest was initiated in order to unravel the secrets of nature, observed and unobserved. Mathematics was called upon to satisfy the queries of how why, when, where, and what and the schools carried on an exhaustive

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102. Cf. ibid., v., 233. Cf. also, Jaina, L. C., Mathematical foundation of Jaina Karma System, op. cit.

102-A. The works on Karma theory, like the Kasāya Pāhuḍa and the Saṭkhandāgama were composed in the c. 1st or 2nd century A. D. Naturally, the works appear to have been founded long before them, and they needed mathematical manoeuvre.

campaign to measure every object, its qualities and events, in various coordinate frames.[103]

Let us denote

Area of a circle as A, d its diameter, r its radius, s as arc of its segment whose height is h, c the chord and p the circumference.

The following relations are found in the bhāṣya of the Tattvārthādhigama-sūtra.[104]

1. $p = \sqrt{10\,d^2}$
2. $c = \sqrt{4\,h\,(d-h)}$
3. $s = \sqrt{6\,h^2 - c^2}$
4. $h = \frac{1}{2}\left(d - \sqrt{d^2 - c^2}\right)$
5. $d = \left(h^2 + \frac{c^2}{4}\right) \div h$
6. $A = \frac{1}{4} \cdot p \cdot d$
7. The part of the circumference of the circle between two parallel chords is half the difference between the corresponding arcs.

All the above relations are also available in Jambūdvīpa samāsa[105] with the exception of (4). It, instead, is

$h = \sqrt{(s^2 - c^2)/6}$

In Gaṇita sārasaṃgraha,[106]

$s\ (\text{gross}) = \sqrt{5\,h^2 + c^2}$

$s\ (\text{fine}) = \sqrt{6\,h^2 + c^2}$

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103. Sūtrakṛtāṅga sūtra mentions, that geometry is the lotus in mathematics, ..., and the rest is inferior. (Śrutaskandha, ch. i, v. 154).

104. Ed. K. P. Mody, Calcutta, 1903.

105. Cf. ibid., appendix (C).

106. Cf. ed. Jain, L.C., Sholapur, 1963, ch. III, vv, 43, 73½.
Cf. also Gupta, R. C., Jaina Formulas for the Arc of a Circular Segment, Jain Journal, 13(3), Jan. 1979, 83-94.

Greek Heron of Alexandria (c. +200) finds[107]

$s = \sqrt{4h^2+c^2} + \frac{1}{4}h$

or $s = \sqrt{4h^2+c^2} + \left\{ \sqrt{4h^2+c^2} - c \right\} \frac{h}{c}$

The Chinese Ch'ēn Huo (1075) gives[108]

$s = c + 2\frac{h^2}{d}$.

The formula (4) requires the solution of a quadratic equation. The Hindu values are older and neater, for Āryabhaṭa II (c. +950) gives

$s \text{ (neat)} = \sqrt{\frac{288}{49}h^2+c^2}$.

Similar formulas occur in Kṣetra samāsa and Laghukṣetrasamāsa.

Except the formula for the volume of frustum of a cone in the Dhavalā text discussed earlier, the following formulae are available in the Tiloyapaṇṇatti :

1. Volume of a right circular cylinder (v.1.116) $= \sqrt{10}\, r^2h$
2. Volume of the wedge shaped universe in the form of a frustum of a right prism (v. 1.165) = Area of the Base × height of prism where Area of the Base =

   $\frac{\text{mukha (mouth)} + \text{bhūmi (ground)}}{2} \Big\}$ perpendicular distance between sides
3. $p = \sqrt{d^2 \times 10}$ (v.4.6)
4. $(\text{Chord of a quadrant arc})^2 = 2r^2$ (v.4.70)
5. $c = \sqrt{4\left[\left(\frac{d}{2}\right)^2 - \left(\frac{d}{2} - h\right)^2\right]}$ [v.4.180]]

   J. P. gives the rule

   $c = \sqrt{4.h\,(d-h)}$ [v.2.23; 6.9]
6. $s = \sqrt{2\,[(d+h)^2 - (d)^2]}$ [v.4.181]

   J. P. gives the rule

   $s = \sqrt{6\,(h)^2+(c)^2}$ [v.2.24,29,6.10]

107. Cf. Health, op. cit. Vol. 2, p. 331.

108. Y. Mikami, The Development of Mathematics in China and Japan, Leipzig, 1913, p. 62.

7. $h = \frac{d}{2} \left[\frac{d^2}{4} - \frac{c^2}{4}\right]^{\frac{1}{2}}$ [v.4.182]

J. P. also gives the rule . [v.2.25, 6.11]

This can be obtained from the quadratic equation
$4h^2 + c^2 - 8r.h = 0$

From (5) and (6) one can obtain $s^2 = 6h^2 + c^2$.

8. Area of segment of a circle (v.4.2374)

$= \frac{hc}{4} \sqrt{10}$

9. Volume of conch figure (v.5.319)

$\left[(\text{Extension})^2 - \left(\frac{\text{mouth}}{2}\right) + \left(\frac{\text{mouth}}{2}\right)^2\right] \times \frac{2}{4}$

Trilokasāra furnishes following formulae :[109]

1. p (gross) $= 3d$ (v.311)
2. p (subtle) $= \sqrt{10}\, d$,,
3. $A = \frac{1}{4}\, pd$,,
4. $r = \frac{9}{16}$ (Side of square of equal area)

   or $\pi = \left(\frac{16}{9}\right)^2$ (v.18)

5. $c^2 = 4h\,(d - h)$
6. $s^2 = 6h^2 + c^2$ } (v.760)
7. $d = \frac{c^2 + 4h^2}{4h}$ (v.761)
8. A (gross) $= \sqrt{10}\; c\, \frac{h}{4}$ (v.762)
9. A (neat) $= \frac{1}{2}\,(c + h)\, h$ (v.762)
10. $d = \frac{c^2 + (2h)^2}{4h}$ (v.763)

109. Cf. also Kapadia, H. R., op. cit., intro, p. XLIII.

11. $c^2=S^2-6h^2$

12. $s^2=4h\left((d+\frac{h}{2}\right)$ } (v.766)

13. $h=\sqrt{(s^2-c^2)\div 6}$ (v.763)

14. $d=\frac{1}{2}\left(\frac{a^2}{2h}-h\right)$ (v.765)

15. $h=\frac{1}{2}\left(d-\sqrt{d^2-c^2}\right)$ (v.764)

16. $h=\sqrt{d^2+\frac{1}{2}s^2}-d$ (v.765)

Similarly Gommaṭsāra supplies us with formulae about volumes of a prism, as base into height. The volume of a sphere is $\frac{9}{2}$ (radius)^3.

A strange problem is posed when one refers Bhagvati (vv.726, 727), dealing with the least number of space-points (pradeśas) required to construct certain figures. This appears to be connected with eight madhya pradeśas which always remain stable in a being, as well as the Descarte frame in which the movement of objects is directed along the world lines.

The least number of odd and even pradeśas make the figures as follows :

Odd	Even	Figure
3	2	Line
9	4	Square
27	8	Cube
15	6	Rectangle
45	12	Parallelopiped
3	6	Triangle
35	4	Triangular Pyramid
5	12	Circle
7	32	Sphere .

(f) Symbolism :

The Jaina School of Mathematics appears to have developed its own symbolism which has been exposed by Ṭoḍarmala[110] in two chapters which he calls Arthasaṃdṛṣṭi (norms of symbolism). One chapter is on the Gommaṭasāra and the other is on the Labdhisāra. There is a long series of

110. op. cit.

symbols for various terms, operations etc. Symbols are numerical, algebraic and geometric.

The ordinals saṃkhyeya, asaṃkhyeya, and ananta were denoted by ड_{e}, a, ख respectively..

The point sets sūcyaṅgula and Jagaśreṇi were denoted by 2 and—, respectively.

The Jīva rāśi, Pudgala rāṣi, Kāla rāśi, Akāśa rāśi, were symbolized as १६, १६ ख, १६ ख ख, १६ ख ख ख respectively.

Thus the trend towards symbolization of norms was carried on to extreme mathematical abstraction. Systematic equations for karma theory were set up through geometric as well as algebraic procedures, quite strange for a modern mathematician. The reader may find the best of the exposition in Ṭoḍaramala's Arthasaṃdṛṣṭis, for historical or relevant studies for interdiscipline purposes.

CHAPTER 6.

TRANSMISSION AND INFLUENCE

In view of the earlier material, a study into the transmissions and influences of mathematieal knowledge, in harmonious phases, between ancient, medieval civilization and Jaina School may be pursued. The period between the–5th century and +5th century is regarded as a dark period in history of Indian mathematics, although the wave of search and effect of the truth, non-violence and work rights appears to be in full swing throughout the world.

The neighbourhood of the Christian era brought with it a scientific awakening through an unprecedented mathematical abstraction at several centres of the world, simultaneously, in the sense, that similar ideas travelled to diametrically opposite parts of the world in an incredibly short span of time.[111]

(a) Problem

The problem, therefore, is concerned with that of origination of a few sources as well as that of the connecting links which joined the mathematico-scientific world through some motivation. Indians developed their philosophies for motivating Satyam Śivam Sundaram, and the Jainas developed their Karma theory for its mathematical study. The set theoretic approach was adopted, and the functional or otherwise structures were allotted measures in symbols through a cosmographic geometry.[112]

(b) Egypt and Jaina School

The question arises, what did the Greeks learn from the prevalent dead mathematics of the Egyptians (in c.—5th century) ? Waerden[113] opines that the Greeks might have learnt their method of multiplication and calculus of fractions. This calculus, however, could not be a basis for

Cf. Needham and Ling, Op. cit, vol. I, 1954, Cambridge, pp. 153-55, & vol.

111. 1959, p, 146 et seq.

112. Cf. T. P., op. cit.

113. Cf. Science Awakening, op. cit.

developing higher algebra. The Greeks developed geometry also independently, but for the Greeks it was simply applied arithmetic. During the Roman period as well, when predictive astrology developed, the astronomy of Egypt remained behind that of Greece and Babylon. However, the comparison between the Jaina School & Egyptian mathematics is as follows:-

1. Not only in the Moscow Papyrus,[114] but in the Rhind Papyrus as well (c.—1700), the value of π is $(\frac{16}{9})^2$ or 3.1605. The same value of π appears in Trilokasāra[115] of Nemicandra (c. +11th century). The other value used in Tiloyapaṇṇattī is $\sqrt{10}$ which resembles the former to two places of decimals.

2. At the sun-temple of Edfu in Egypt (—100), the formula used for finding area of trapeziums is the same as that used in Tiloyapaṇṇattī.[116]

3. One more value of π used by Egyptians[117] is 3.2 which appears to be the same as that used by Vīrasena[118] in Dhavalā as $\frac{355}{113}$.

4. Rajju[119], a cosmological measure in Jaina texts appears in Egypt[120] as the rules of the rope in the ratio of 3: 4: 5.

5. The method of duplatio and mediatio in Egypt[121] appears in Jaina texts as squaring and ardhaccheda or vargaśalākā extraction operations. The rules of false position of Egypt appear in Mahāvīra[122] and elsewhere.

6. Virasena uses formulae for finding volumes of frusta of pyramids on square base, as appear in Egypt[123].

(c) Greece and Jaina School

Neugebauer, Becker and Reidemeister, finding that algebra does not

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114. Cf. Coolidge, J. L., A History of Geometrical Methods, Oxford, (1940), p. 11. These values for π have been in use from ancient times to the medieval period, and suggest problems of the source, when they appear simultaneously in different civilizations far apart.
115. V. 18, op. cit. Cf-also other texts of Karaṇānuyoga.
116. VV. 1-165, 181. Cf. also Ṣaṭkhaṇḍāgama, Book 4, v.v. 1, 3 etc.
117. Heath, op. cit, vol. 1, p. 125.
118. Ṣaṭkhaṇḍāgama, book—4, v. 14, p. 40.
119. T. P. G., pp. 99–101.
120. Waerden, op. cit., p. 6.
121. Ibid., p. 18.
122. Gaṇitaśāra saṃgraha.
123. Cf. Waerden, op. cit., pp. 34–35; Cf. also T. P. G., pp-25, et seq.

start from Diophantus, but quite earlier from Mesopotamia (c.—2000), and as such the Pythagorean arithmetica deserves to be Babylonian arithmetica. Thus Waerden[124] has tried to prove :

1. Thales and Pythagoras started with Babylonian mathematics and gave it a special character, quite different from it.
2. Mathematics was developed in advanced forms in the Pythagorean schools and outside, and began to satisfy the problems of logic.

He has also supported the following hypothesis in astronomy[125], "Freudenthal's hypothesis reduces therefore to the following : Before becoming subject to the Greek influence, the Hindus had a versified positional system, arranged decimally and starting with the lowest units. They had the digits 1-9 and similar symbols for 10, 20,......Along with Greek astronomy, the Hindus became acquainted with the sexagesimal system and the zero. They amalgamated this positional system with their own; to their own Brahmin digits 1–9, they adjoined the Greek 0 and they adopted the Greek Babylonian order."

Regarding the Buddhist's use of large numbers, he remarks,[126]

"It is clear that these numerals were never used for actual counting or calculations. They are pure fantasies which like Indian towers, were constructed in stages to dazzling heights."

The attention of above scholars may be attracted towards the source material of Prakrit texts like the Ṣaṭkhaṇḍāgama and the Tiloyapaṇṇattī, as also the Śvetambara texts, wherein the motivation to use the very large numbers is evident. Therein the basis of space and time measures is principle-theoretic and not construction—theoretic as in Greek works of B.C. The exposition of the Jaina Karma theory needed the infinities in space and time, apart from the innumerables and numerables.[127] It also needed various types of set-thoretic measures in simile (equivalence), based on relations between sets themselves, and as such apart from their other discoveries, necessity might have led them to formation of place value systems elaborated earlier.

The following facts are also important to relate :

1. In the Tiloyapaṇṇattī, a formula about the chord belongs to Babylon, recorded at —2600 . The formula for the arc is,

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124. Cf. p. 5, op. cit.
125. Cf. ibid., pp. 56-57.
126. Cf. ibid., p. 52.
127. Cf. research paper of Jain, op. cit. Cf. also, T.P.G.

however based on the value of π as $\sqrt{10}$, which is not available in Babylon.[128]

2. Virasen has made use of the method of application of areas prevalent in Babylon and Greece, for analysing the measures of transfinite sets.[129]

3. In the Tiloyapaṇṇattī (or any other text of the Karaṇānuyoga group) Babylonion & Greek sexagesimal measure appears as basic to a complete revolution in 60 muhūrtas for projected paths of real and counter bodies. "Sixty" is a factor of the 109800 celestial parts.[130]

To Thales was ascribed the saying, "Know thyself".[131] He predicted the solar eclipse of –585 and that he could draw upon the experience of Oriental astronomers.[132]

Thales also tried to prove (1) A diameter divides a circle into two equal parts. (2) The base angles of an isoscles triangle are equal. (3) Equal angles are obtained from two intersecting straight lines.[133] Needless to say, that this geometry was basic to Jaina cosmography. Pythagoras is said to have discovered the theory of irrationals[134] which were used in set-theoretic approach of the measures of bio-cosmos of the Jainas. The Phythagoreans[135] regarded the planets as living gods and based the lunar calculations on 59.

It is important to note that China and India made use of the shifting of lunar mansions in a period of about 1000 years, (about—1400 and —350), which agree with those in the Vedāṅga Jyotiṣa and the Tiloyapaṇṇattī.[136] They had also recognised the change of seasons with the precession of equinoxes as the Greeks.

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128. Cf. Coolidge, op. cit., pp. 6, 7.
129. Cf. Ṣaṭkhaṇḍāgama, book-3, pp. 42–43.
130. Cf. Jain, L. C., The Kinematic Motion of astral real and counter bodies in Trilokasāra, I.J.H.S., vol. 11, no. 1, 1976, 58–74.
131. Cf. Waerden, p. 85.
132. Such a theory of astronomy was perfected in Jaina School, round about the same period. Cf. Waerden, p, 86., op. cit.
133. Cf. ibid., p. 89.
134. Cf. ibid., p. 90.
135. Cf. Olmsteed, A.T., History of Persian Empire, Chicago, 1948, p. 209, Cf. also Bibliography (A) of source material.
136. Cf. Shastri, research bibliography.

Like the Pythagoreans, the Jainas made the irrationals observable through geometry and discussed the poly-ended views at large. The paradoxes of Zeno of Elea could be solved through the similar logics and theories of the Phythagoreans and the Jainas.[137]

(d) China and Jaina School

Waerden notes, "The oldest Chinese collection of problems on applied proportion[1] looks like an ancient Babylonian text, but it is next to impossible to prove their dependence or to trace the road along which they were transmitted "[138] "It is rather a general impression of relatedness which makes itself felt when one knows the cuneiform texts and then looks through Heron or Diophantus, or the Aryabhatiya of Aryabhata or the Algebra of Alkhwarizmi "[139]

The same impression will be felt when one knows the Prakrit texts, specially the Artha Saṃdṛṣṭis and the latter. The similar effect will be felt about the paradoxes of Hui Shin and those of the Eleatic Zeno, when compared with the relativitism and polyends theory of the Jaina texts, alongwith their basic mathematical concepts of space and time, attributes and entities.

The rule for the area of segment of a circle and other survey material given in the Chiu Chang Suan Shu (Nine Chapters on the Mathematical Art) of the —1st century reappear in the +9th century work of Mahāvīra,[140] comparable also with the results in the Tiloyapaṇṇattī and the Trilokasāra, or in any other text of the Karaṇānuyoga group.

In China[141], square roots and cuberoots were developed in the +1st century, whereas in the Ṣaṭkhaṇḍāgama, they appear in +2nd century and go upto the twelfth roots of sets needed for philosophical details.[142] The rule of three is found in Han Chiu Chang[143], earlier than in any other Sanskrit text, but the set theoretic contexts (trairāśika, etc.) in Prakrit texts is earlier than Christian era, and resembles the words Shih and Fa of China[144] with those of Pramāṇa, Icchā and Phala in India.[145]

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137. Cf. Gaṇita Sāra Saṃgraha, op. cit., intro. p. 6, et seq.
138. "Science Awakening", p. 278.
139. Cf. ibid. 280.
140. Cf. Needham and Ling, vol. I, op. cit., p. 155.
141. Cf. ibid, vol. III, op. cit., pp. 65–68.
142. Book, 3, vv. 1, 2, 17. Cf. also Jain, research papers bibliography (C), on set theory etc.
143. Cf. Needham and Ling, vol. III, pp. 26, 35, 146.
144. Cf. ibid., pp. 65, 146.
145. Cf. Jain, Set theory...etc., Bib. (C).

In the post-universal measures (Jaina texts), fractions are written in vertical column as had been the practice in China where Han counting-boards were utilized.[146] The Dhavalā commentaries of the +9th century manipulate several types of fractions as described by Singh.[147] In China[148], the negative numbers appear in +1st century, where as this was in terms of subtraction of sets in Jaina School. From the records in the Tiloyapaṇṇattī and the Dhavalā texts, the word Ṛṇa (Riṇa) with symbols, O, +, ᶜ etc. was used for mathematical purposes.[149] The sense of the past carried the concept of negative infinity, and the anti-affine levels to infinity signified the negatives.

The theorem of Pythagoras is used in several contexts in the Karaṇānuyoga texts.[150] The fundamental identity of the algebraic and geometrical relations mentioned by the Chinese[151] throughout the previous millennium may also be seen in the Dhavlā texts in philosophical contexts.[152] The method of application of areas elsewhere is to be seen in the work of Al-Khwarizmi (+9th century) in Persia.

The rule of false position in the Prakrit texts[153] in form of slightly small or slightly great, appear in the text of Chiu Chang Suan Chu of the —1st century.[154] Indeterminate analysis[155] appear to be in full swing in China in +4th century. The Ta Yen method of the Chinese was known in India as Kuṭṭaka. The Hundered Fowls problem in Chang Chhin-Chien Suan Ching (+4th century) reappears in Mahavīra's work.

The study of pyramidal frusta in China in the +1st century resemble to appreciable extent with those in the Tiloyapaṇṇattī.[156]

The following three principles of analytical geometry were utilized

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146. Cf. Needham and Ling, vol. III, pp. 81, 146.
147. Cf. Bibl. (C), 12(a), & 5(d).
148. Cf. Needham and Ling, pp. 26, 43, 45, 90, 130. 146.
149. Cf. Bibl (c), 6 (c).
150. Cf. T.P.G.
151. Cf. Needham & Ling, vol. III, p. 147, 107.
152. Cf. Bibl (c), 5 (b) and (d).
153. Ṣaṭkhaṇḍāgama, book—4, v. 1. 2, 51.
154. Cf. Needham & Ling. vol. III, p. 147.
155. Cf. ibid., pp. 147, In Prakrit texts it is used in details on topics of Karma theory. Cf. Gommaṭasāra, and Labdhisāra, op. cit.
156. Cf. ibid. p. 97.

in China[157] and the Jaina School.[158] (i) The invention of systems of coordinates, (ii) Recognition of the method of correspondence, (ii) the existence of functional relations between two variables.

Similar to Chinese arithmetica and combinatorial analysis, the Jaina School had its own arithmetica and the method of combinatorial analysis round about the Christian era.[159] The number mysticism of the Pythagorean era appears to be a part of what was elaborately having studied in India. Artha, in the Jaina School, meant the measure of objects with respect to the Fluent (Dravya), Quarters (Kṣetra), Time (Kāla) and Phase (Bhāva). The magic squares and other geometrical forms could also be seen in the Jaina School.[160]

Analogous to the system of large numbers in Jain School, in the texts of the Chou date, (—6th to +4th century—Confucian period), several terms for very large number appear.[161] The Chinese like the Zeno and the Jainas, found the impossibility of divisibility ad infinitum in space and time,[162] The Chinese had also developed the naive ideas of matrices comparable to those in Jaina School.[163]

(e) Conclusion

From the above, it could be concluded that the Jaina School of Mathematics formed one of sources of transmission and influence. The religio-philosophic garb, however, did not allow a deeper probe into the evolution of the scientific spirit which was originated and motivated at the time of Vardhamāna Mahāvīra, and which seems to have been based on some hidden under-current.

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157. Cf. ibid. p. 106.
158. Sarvārthasiddhi of Pūjyāpāda, op. cit, pp. 185 et seq; ch. 2, vv. 28, 29, 30; ch. 7, vv. 36, 39, 65, 68, 83, 89, 93, 96, 99, 104, 108, 117, 161, 178, 189, 186, 216-17, 228, 237, 246, 265, 272, 276. Cf. also research papers in Bibl. (c) by Jain.
159. Cf. Datta, Bibl. (c).
160. Cf. Bibl. (c), 14.
161. Cf. Needham & Ling, vol. III, pp. 82, et seq.
162. Cf. ibid., p. 92.
163. Cf. ibid., p. 117. Cf. also T.P.G. (Tiloyapaṇṇatti Kā Gaṇita, op. cit.)

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राजस्थान प्राकृत भारती संस्थान, जयपुर

—अद्यावधि प्रकाशित ग्रन्थ—

1.	कल्पसूत्र सचित्र	(मूल, हिन्दी एवं अंग्रेजी अनुवाद तथा 36 बहुरंगी चित्रों सहित) सम्पादक एवं हिन्दी अनुवादक: **महोपाध्याय विनयसागर**; अंग्रेजी अनुवादक: **डा० मुकुन्द लाठ**	200-00
2.	राजस्थान का जैन साहित्य	(राजस्थानी विद्वानों द्वारा रचित प्राकृत, संस्कृत, अपभ्रंश, राजस्थानी, हिन्दी भाषा के ग्रंथों पर विविध विद्वानों के वैशिष्ट्य पूर्ण एवं सारगर्भित 36 लेखों का संग्रह)	30 00
3.	प्राकृत स्वयं शिक्षक	लेखक—**डा० प्रेमसुमन जैन**	15-00
4.	आगम तीर्थ	(आगमिक प्राकृत गाथाओं का हिन्दी पद्यानुवाद) अनु० **डा० हरिराम आचार्य**	10-00
5.	स्मरण कला	(अवधान कला सम्बन्धित पं० धीरजलाल टो० शाह लिखित गुजराती पुस्तक का हिन्दी अनुवाद) अनु० **मोहन मुनि शार्दूल**	15 00
6.	जैनागम दिग्दर्शन	(45 जैनागमों का संक्षिप्त परिचय) सजिल्द ले० **डा० मुनि श्री नगराजजी** सामान्य	20-00 16-00
7.	जैन कहानियाँ	ले० **उपाध्याय महेन्द्र मुनि**	4-00
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9.	हाफ ए टैल (अर्धकथानक)	(कवि बनारसीदास रचित स्वात्मकथा अर्धकथानक का अंग्रेजी भाषा में अनुवाद, आलोचनात्मक अध्ययन एवं रेखा चित्रों सहित) सम्पादक एवं अनुवादक: **डा० मुकुन्द लाठ**	150-00
10.	गणधरवाद	(दलसुखभाई मालवणिया लिखित गुजराती गणधरवाद का हिन्दी अनुवाद) अनु० **प्रो० पृथ्वीराज जैन** सम्पादक—**महोपाध्याय विनयसागर**	50-00

— मुद्रणाधीन ग्रन्थ —

1.	जैन इन्सक्रिप्सन आफ द राजस्थान	(राजस्थान के प्राचीन, ऐतिहासिक एवं वैशिष्ट्य पूर्ण जैन शिलालेखों, मूर्तिलेखों का परिचयात्मक वर्णन) ले० **रामवल्लभ सोमानी**
2.	एन्जेक्ट सायन्स फ्रोम जैन सोर्सेज पार्ट I, बेसिक मेथेमेटिक्स	ले० **लक्ष्मीचन्द जैन**
3.	उपमिति भव प्रपंचा कथा	(महर्षि सिद्धर्षि रचित ग्रन्थ का हिन्दी अनुवाद सं० एवं अनु० **महोपाध्याय विनयसागर** तथा अनु० **लालचन्द जैन**
4.	अपभ्रंश और हिन्दी	**डॉ० देवेन्द्रकुमार जैन**
5.	बौद्ध एवं गीता के आचार दर्शन के संदर्भ में जैन आचार दर्शन का तुलनात्मक एवं समालोचनात्मक अध्ययन	**डॉ० सागरमल जैन**

सम्पादनाधीन ग्रन्थ

1.	ऋषिभाषित सूत्र	(हिन्दू, बौद्ध और जैन सर्वज्ञ ऋषियों के सारगर्भित उद्बोधन; मूल हिन्दी एवं अंग्रेजी अनुवाद) अनु० **महोपाध्याय विनयसागर; कलानाथ शास्त्री**
2	नीतिवाक्यामृत	(आचार्य सोमदेव रचित राजनीति के सिद्धान्तों का हिन्दी व अंग्रेजी में अनुवाद) अनु० **डॉ० एस० के० गुप्ता; डा० बी० आर० मेहता**
4.	गाथा सप्तशती	(हाल कवि रचित सप्तशती का हिन्दी व अंग्रेजी अनुवाद) अनु० **डॉ० हरिराम आचार्य; डी० सी० शर्मा**

4.	एन्जेक्ट सायन्स फ्रोम जैन पार्ट–II कोस्मोलोजी एण्ड एस्टोनोमी सोर्सेज	ले० लक्ष्मीचन्द जैन ,,
5.	पार्ट–III सिस्टमथियरी	,,
6.	पार्ट–IV सेट थियरी	
7.	पार्ट–V थियरी आफ अल्टीमेट पार्टीकल्स	,,
8.	त्रिलोकसार	नेमिचन्द्राचार्य रचित ग्रन्थ का हिन्दी एवं अंग्रेजी अनुवाद) अनु० लक्ष्मीचन्द जैन
9.	जैन साहित्य का संक्षिप्त इतिहास	(स्व० मोहनलाल दलीचन्द देशाई लिखित 'जैन साहित्य नो संक्षिप्त इतिहास' गुजराती का हिन्दी अनुवाद) अनु० कस्तूरचन्द बांठिया
10.	एपीटोमी आफ जैनिज्म	स्व० पूरणचन्द्र नाहर
11.	मथुरा के जैन शिलालेख	,, ,, ,,
12.	स्टडीज् आफ जैनिज्म	ह० टी० जी० कलधटगी
13.	धातुपरीक्षा	(ठक्कुर फेरु रचित ग्रन्थ का हिन्दी एवं अंग्रेजी अनुवाद) अनु० डॉ० धर्मेन्द्रकुमार
14.	प्रतिष्ठा लेख संग्रह द्वितीय भाग	महोपाध्याय विनयसागर
15.	श्रीवल्लभीय राजस्थानी संस्कृत शब्दकोष	,, ,,
16.	प्राकृत काव्य मंजरी	
17.	प्राकृत शब्द सोपान	
18.	प्राकृत संज्ञा एवं सर्वनाम प्रकरण	डॉ० उदयचन्द जैन
19.	वज्जालग्ग में जीवन मूल्य	भाग–1 डॉ० कमलचन्द सोगाणी
20.	,, ,, ,,	भाग– ,, ,,
21.	वाक्पतिराज की लोकानुभूति	,, ,,
22.	भगवान महावीर: जीवन और उपदेश	

23. जैन दर्शन की रूपरेखा डा० कमलचन्द सोगाणी

24. जैन संघ की परम्परा और विकास

25. जैन कला की भूमिका

26. प्राकृत साहित्य : एक परिचय

27. अपभ्रंश साहित्य : एक परिचय

28. संस्कृत का जैन साहित्य

29. राजस्थानी जैन साहित्य

30. राजस्थान के प्रमुख जैन ग्रन्थ भण्डार

31. जैन धर्म और समाज

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1. एक हजार रुपये से अधिक प्रकाशन खरीदने पर 40% कमीशन और संस्थान के प्रकाशनों का पूरा सेट खरीदने पर 30% दिया जाता है।
2. डाक-व्यय एवं पैकिंग व्यय पृथक् से होगा।

प्राप्ति स्थान :

राजस्थान प्राकृत भारती संस्थान,
यति श्यामलालजी का उपासरा,
मोतीसिंह भोमियों का रास्ता, जयपुर-3
पिन कोड–302 003
